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Measurement of beating effects in narrowband multimode Lamb wave displacement fields in aluminum plates by pulsed TV holography

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ABSTRACT

Narrowband ultrasonic surface acoustic waves are of the greatest current interest for the nondestructive testing of thin-walled members and shell structures like plates, pipes, bridge girders, cans and many others. The measurement and characterization of ultrasonic displacement fields of Lamb waves by pulsed TV holography (TVH) is presented. Narrowband ultrasound is generated in a few millimeters thick aluminum plate by the prismatic coupling block method using a tone-burst excitation signal in the range of 1MHz. At this frequency, the plate supports only a few Lamb wave modes, mainly the A0 and S0 ones. The simultaneous presence of these modes produces a beating clearly detectable as a spatial amplitude modulation. Our self-developed TVH system performs the optical phase evaluation by the Spatial Fourier Transform Method and renders the instantaneous out-of-plane mechanical displacement field along the whole inspected area. From this field, the wavenumber of each Lamb mode can be obtained and, by combining them with the value of the ultrasound frequency and with the Rayleigh-Lamb theoretical frequency spectrum, information about the elastic constants of the specimen material is obtained.

Keywords: Ultrasonic Field, Lamb Wave, Surface Acoustic Wave, TV Holography, Electronic Speckle Pattern Interferometry.

1. INTRODUCTION

Acoustic phenomena in general provide very adequate means to obtain information about the elastic characteristics of media and also for non-destructive testing. The elastic constants are closely related to the velocity of the acoustic waves that are different depending on the type of wave. In an isotropic, homogeneous (bulk) medium, only two types can exist: dilatational or longitudinal waves and distortional or shear waves, with phase velocities $c_L$ and $c_T$ respectively. However, if there is lack of homogeneity (for example, bounds between different media), then new acoustic modes appear, each with a particular dispersion curve in which, even in the simplest media, both phase velocity $c_p$ and group velocity $c_g$ are in the majority of cases frequency-dependent due to geometrical constraints imposed to the possible solutions.$^{1,2}$

The existence of layers of different materials allows the propagation of guided waves. One of the main attractiveness of the guided waves is their capability to propagate along large distances with much less attenuation than bulk waves. Another is that the field distribution is mode dependent (for example, in some modes the amplitude of the displacements decays with the depth into the medium), which allows to improve the performances of a given measurement method by a proper selection of the acoustic mode and frequency.$^{3,4}$ However, the difficulties for generating only a single mode and the dispersive nature of the guided waves make necessary in many cases to deal with complicated signals that require a careful data processing. In fact, one of the key subjects for the usefulness of guided waves is the isolation of each mode contribution to the whole signal and, within each mode, to take into account the dispersive effects. Broadband signals contain more information along their bandwidth than narrowband signals but the dispersive effects may be minimized and the signal to noise ratio maximized in the last ones, being in consequence more precise for the determination of phase or group velocities.

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Lamb waves are a kind of acoustic guided waves that propagate along plates with stress-free boundary conditions. Their application to non-destructive testing was already described in the 1950s and since then a considerable number of researchers have demonstrated different applications for testing and for determining the elastic properties of materials. Single- or multimode Lamb waves, each with broad or narrow frequency spectrum, can be generated.

To extract information from broadband (single- or multimode) signals, time-frequency analysis techniques are adequate. For example, the wavelet transform (WT) has been employed to identify Lamb modes and to extract group velocity dispersion curves. Other time-frequency methods include the short-time Fourier transform and the Wigner distribution. A different approach that also allows discriminating modes in broadband signals is based on two-dimensional spectra obtained by combining time records from a series of equally spaced points along the plate, rendering the amplitude of the signal versus frequency and wavenumber. The knowledge of the wavenumber at a given circular frequency allows to calculate the phase velocity value. Other technique allows to extract the phase velocity in a broad spectrum for several modes from the transient time histories of the waveforms recorded at two locations. For single-mode signals, a classical phase velocity measurement technique consists of determining the phase shift of the signal at each frequency due to a known displacement of the receiver. Other method of broadband analysis is based on the cut-off frequencies of leaky Lamb waves.

For narrowband signals, the signal processing is simpler than for broadband ones. In most of the cases the group velocity dispersion in the band of interest can be neglected and the shape of the signal is preserved along the propagation distance. If the signal is also single-mode, the phase velocity may be measured in a simple way by the determination of the phase shift over a known path length. The most common difficulties arise from the simultaneous presence of several modes. In this case, an easy way to separate them is by locating the monitoring point far enough from the excitation point to allow the envelopes of the modes to be completely separated from each other. This method works fairly well, provided there is room enough in the inspected part to allow the separation of the envelopes, for which the excitation should be in the form of a not very long tone-burst. Otherwise, some of the aforementioned techniques for broadband signals (for example, time-frequency analysis techniques or two-dimensional Fourier transform) may be applied to separate the existing modes, at the expense of increasing the computational work. The phase velocity can be obtained, at each frequency, as the quotient between the center circular frequency and the center wavenumber of a sufficiently narrow wave group.

Due to the intrinsic capability of the narrowband signals to render more precise results than the broadband ones, they are adequate to the evaluation of the bulk wave velocities and . The center frequency may be measured at any fixed location by Fourier transforming the time domain signal from a point (single-channel) probe (e.g., piezoelectric transducer, laser velocimeter, EMAT, optical beam deflection probe, etc.). On the other hand, the acquisition of spatial information and specifically the center wavenumber is not so straightforward because it requires performing a spatial sampling. With point detectors, the problem is solved by the mechanical scanning of the probe along the wave propagation path, but the number of sampled points is usually small due to experimental limitations. This leads to a low resolution in the calculated values of the spatial frequencies.

An alternative approach is the employment of whole-field optical techniques to record the acoustic displacement field over a finite area with high spatial resolution. In the last years, the increased capability of TV holography (TVH) and holographic interferometry (HI) techniques opened new possibilities for the analysis of ultrasound, mainly with sensitivity to out-of-plane displacements. Among the advantages of holographic techniques we can highlight:

- True remote operation (standoff of several m).
- No alteration of the phenomenon to measure.
- Bidimensional acquisition array with high lateral resolution and selectable field of view over a wide range.
- High measurement speed with applicability to moving objects.
- Possibility to measure inside vacuum chambers, furnaces, parts with restricted accessibility (tanks, framework, etc.), high temperature or lightweight parts, and in other tasks not feasible by traditional ultrasonic techniques.
In both technologies, HI and TVH, there are basically two approaches: stroboscopic and pulsed. The first is only valid to study stationary phenomena but allows a good sensitivity (subnanometer level)\(^{19}\) while the second allows, due to its short acquisition time, to perform the analysis of transients with fine temporal resolution, high immunity to environmental perturbations and flexibility to select the acquisition times close enough to the excitation to avoid the need of acoustic absorbers at the boundaries of the inspected part. However, its sensitivity is slightly worst (of the order of one nanometer). Our group contributed to the demonstration of the detection of Rayleigh and Lamb waves by pulsed TVH\(^{20,21}\) and recently developed a refinement of the technique that, in a first stage, performs the optical phase evaluation and, subsequently, the mechanical complex amplitude evaluation\(^{22}\). This measurement tool allows “freezing” and recording the instantaneous spatial distribution of transient out-of-plane displacement fields.

In the case that two narrowband modes of similar out-of-plane amplitudes at the free surface and similar center frequency values coexist in the same region of the plate beating effects appear. These consist in a spatial modulation of the displacement amplitude, with nodal lines (amplitude minima) periodically located along the propagation direction with a beat wavelength that corresponds to a 2\(\pi\) phase difference between interfering modes. This behavior was already theoretically and experimentally described. For example, quasi-Rayleigh waves can be generated in a plate by superposition of Lamb modes A0 and S0.\(^1\) Due to the different phase velocity of each mode, a phase difference between them is accumulated along the propagation path that causes the successive change of location of the perturbation from one plate surface to another. A similar beating phenomenon has been described in a piezoelectric plate.\(^{23}\) However, to the best of our knowledge, up to now there were not available experimental data of the spatial distribution of instantaneous Lamb wave fields with beating effects.

In this work we present bidimensional instantaneous transient out-of-plane displacement fields of Lamb waves in aluminum plates, measured by pulsed TVH. The experimental system is arranged to generate mainly two modes, A0 and S0, in a narrow frequency band, having both modes the same center frequency but different wavenumbers, so that intermode beating effects are clearly visible. Apart from the intrinsic interest of the raw images, the beat wavelength is measured and used to calculate the difference between the wavenumbers of the interfering modes. Also, the center frequency of the modes is measured with a Michelson speckle interferometer. Fitting these two experimental values to the Rayleigh-Lamb theoretical frequency spectrum, supposed known the Poisson’s ratio \(\nu\) of the material, then the working point of each mode can be located in the frequency spectrum and the two bulk wave velocities, \(c_L\) and \(c_T\), can be calculated. From these and from the material density, the elastic constants are derived. In the case that the ratio \(\nu\) was unknown, the former procedure gives a numerical relationship between \(c_T\) and \(\nu\) that can be used to calculate \(c_T\) if another independent constant of the material (for example, the dilatational wave phase velocity, \(c_L\)), is known.

\section{Theory}

\subsection{Lamb waves}

The geometry and notation for the Lamb waves are depicted in Fig. 1. We will suppose that the plate material, with regard to its mechanical response, is linear, homogeneous and isotropic and that it extends indefinitely in the directions \(x_1\) and \(x_3\). Also, we will suppose that the wavefronts on the observed surface \(x_2 = h\) are straight and parallel to the \(x_3\) axis.

The behavior of the envelope of each mode is related to the group velocity, while the behavior of the carrier is related to the phase velocity. If the best accuracy is desired in the phase velocity measurement, it seems convenient to avoid any modulation of the ultrasonic field, so we excited tone-bursts-with constant amplitude and no phase modulation, long enough to suppose that, in each mode, the displacement at each given point is sinusoidal with an amplitude independent of time (with the exception of the zones at the extremes of the wave train where the amplitudes decay to zero). As we will detail in the following section, we have experimentally verified that the frequencies of all the existing modes are identical to the excitation frequency. No reflected waves are present because the acquisition is done just after the burst generation, before the wave reaches any edge of the plate.

We know the nominal values of the mass density \(\rho_{\text{nom}} = 2790\text{kg.m}^{-3}\), of the Poisson’s ratio, \(\nu_{\text{nom}} = 0.33\), and of the Young’s modulus, \(E_{\text{nom}} = 70,000\text{GPa}\), of the aluminum alloy utilized (EN AW-2017A), from which the bulk wave nominal phase velocities, \(c_{L_{\text{nom}}} = 6100\text{m.s}^{-1}\) and \(c_{T_{\text{nom}}} = 3070\text{m.s}^{-1}\), are readily calculated employing (11), (12) and (18). So it is possible to know which modes are feasible to be excited for a given plate thickness \(2h\) and a given frequency \(f\).\(^7\) For the
symmetric modes,

\[ N_S = 1 + \mathcal{E} \left( \frac{2hf}{c_T} \right) + \mathcal{E} \left( \frac{2hf}{c_L} + \frac{1}{2} \right) \]  

(1)

and, for the antisymmetric modes,

\[ N_A = 1 + \mathcal{E} \left( \frac{2hf}{c_L} \right) + \mathcal{E} \left( \frac{2hf}{c_T} + \frac{1}{2} \right) \]  

(2)

The application of (1) and (2) to the frequency \( f = 1.00 \text{MHz} \) and the thicknesses employed, 3, 4 and 5mm, gives the results included in Table 1. Although a considerable number of modes could be excited, the ultrasound generator employed for our experiments is selective enough to excite only the lowest order modes AO and SO.

With the former considerations, we can write the expressions of the instantaneous out-of-plane displacements in the AO and SO modes on the plate free surface as follows:

\[
\begin{align*}
    u_{2\text{SO}}(x_1, h, t) &= u_{2\text{SO}}(x_1) \cos(\varphi_{\text{MSO}} + k_{\text{ISO}}x_1 - \alpha \xi) \\
    u_{2\text{AO}}(x_1, h, t) &= u_{2\text{AO}}(x_1) \cos(\varphi_{\text{MAO}} + k_{\text{IAO}}x_1 - \alpha \xi)
\end{align*}
\]

where \( \varphi_{\text{MSO}} \) is the initial phase and \( k_1 \) the Lamb wavenumber.

The beating can be explained simply by adding both displacements:

\[
\begin{align*}
    u_2(x_1, h, t) &= u_{2\text{SO}}(x_1) \cos(\varphi_{\text{MSO}} + k_{\text{ISO}}x_1 - \alpha \xi) + u_{2\text{AO}}(x_1) \cos(\varphi_{\text{MAO}} + k_{\text{IAO}}x_1 - \alpha \xi)
\end{align*}
\]

from which the beat wavelength \( \lambda \) is given by the condition:

\[
|k_{\text{ISO}}(x_1 + A) - k_{\text{IAO}}(x_1 + A) - (k_{\text{ISO}}x_1 - k_{\text{IAO}}x_1)| = 2\pi
\]

(6)

that is,

\[
|k_{\text{IAO}} - k_{\text{ISO}}| = \frac{2\pi}{\lambda}.
\]

(7)

If the two amplitudes are not very different, a distinct carrier frequency can be observed, whose wavelength \( \lambda_c \) verifies approximately
\[
k_{1A0} + k_{1S0} = \frac{2\pi}{\lambda_c}.
\]

So, if \( A \) is measured from the TVH images, the wavenumber difference can be calculated. If \( \lambda_c \) is also measured, the individual wavenumbers \( k_{1S0} \) and \( k_{1A0} \) can be estimated approximately.

To fit the former data into the theoretical Rayleigh-Lamb frequency spectrum, it is convenient to employ normalized magnitudes:

- Normalized wavenumber: \( \xi = \frac{2hk}{\pi} \) (9)
- Normalized frequency: \( \Omega = \frac{2h\omega}{\pi c_T} \) (10)
- Velocity ratio: \( \chi = \frac{c_L}{c_T} \) (11)

In linear Elasticity, Poisson’s ratio \( \nu \) is a function of the velocity ratio:

\[
\nu = \frac{\chi^2 - 2}{2(\chi^2 - 1)}.
\]

The zones of the frequency spectrum of interest for our problem are the following (Fig. 2):

Zone I, \( 0 < \Omega < \xi \)

\[
F_{1,\text{sym}}^{\text{ant}} = \frac{\tanh \frac{\pi}{2}\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}} - \frac{4\xi^2}{\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}}\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}}} \left( 2\xi^2 - \Omega^2 \right)^2}{\left( 2\xi^2 - \Omega^2 \right)^2} = 0.
\]

Zone II, \( \xi < \Omega < \chi\xi \)

\[
F_{II,\text{sym}}^{\text{ant}} = \frac{\tan \frac{\pi}{2}\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}} + \frac{4\xi^2}{\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}}\sqrt{\xi^2 - \frac{\Omega^2}{\chi^2}}} \left( 2\xi^2 - \Omega^2 \right)^2}{\left( 2\xi^2 - \Omega^2 \right)^2} = 0.
\]

The mode A0 is always in the zone I. The mode S0 is in the zone II in the frequency range from zero to a value \( \Omega_{S0} \) where it crosses to the zone I and remains in this zone for any frequency higher.

The equations (13) and (14) can be solved only numerically. We have devised the following procedure:

a) Taking in account the typical values of the Poisson’s ratio for the aluminum, a conservative range of values of \( \chi \) has been selected:

\[
1.7 < \chi < 2.3 \quad \text{that is equivalent to} \quad 0.235 < \nu < 0.383.
\]

b) From the experimental values of \( A \) and \( \lambda_c \), the individual wavenumbers \( k_{1S0} \) and \( k_{1A0} \) are calculated from (7) and (8) and, with the thickness value \( 2h \), their normalized values \( \xi_{S0}, \xi_{A0} \) are obtained.

c) The functions \( F_1 \) and \( F_2 \) are represented versus \( \Omega \) for different values of \( \chi \) and the fixed values of \( \xi_{S0}, \xi_{A0} \) obtained in the point b). The simultaneous null values of both functions give the possible values of \( \Omega \) and \( \chi \). If there are not coincidences, the normalized wavenumbers are slightly changed maintaining their difference, because the equation (7) is more reliable than (8).
Even considering exact the normalized wavenumber difference, this procedure does not define a unique working point but, instead, a value of the normalized frequency $\Omega$ for each value of the velocity ratio $\chi$ in the full range of values of $\chi$. If the Poisson’s ratio of the material is known, $\chi$ is also known from (12), the procedure is considerably simplified and the values of the normalized frequency and of the normalized individual wavenumbers are rapidly obtained. Alternatively, if the dilatational wave phase velocity, $c_{\text{d}}$, is known, from (10) and (11) a new relationship between $\Omega$ and $\chi$ is established, that allows to resolve the indetermination.

### 2.2. TV holography

The measurement principle of the TVH is based on the recording by means of a TV camera of the interference pattern between a light reference beam and an object beam, reflected from the part under inspection. Image-forming optics allows to relate each pixel on the TV target to a particular point on the inspected surface. We employed a self-developed pulsed TVH system (Fig. 3) that allows the phase evaluation by the Spatial Fourier Transform Method. The system renders the spatial distribution of the optical phase difference, $\Delta\phi$, between two states defined by the respective laser pulses. If the surface motion is harmonic and the time interval between the pulses is an odd number of half-periods (three in our case), then $\Delta\phi$ is proportional to the incremental out-of-plane displacement between both states:

$$\Delta\phi(x_1,t_1) = -\frac{4\pi}{\lambda} [u_2(x_1,h,t_2) - u_2(x_1,h,t_1)] = \frac{8\pi}{\lambda} u_2(x_1,h,t_1)$$

being $\lambda$ the laser wavelength and

$$t_2 = t_1 + \frac{3\pi}{\omega} .$$

A brief description of the particular system that we employed is given in the following section. A more detailed description may be found elsewhere.21,22

### 2.3. Speckle point interferometry

The measurement principle of the speckle point interferometry is based on the single-channel detection of the interference pattern between a light reference beam and an object beam, reflected from a sufficiently small area of the
part under inspection.\textsuperscript{25} We employ a Michelson arrangement with the object beam focused onto the inspected surface by a lens (in our case, a microscope objective) in such a way that there are a small number of speckle grains in the reflected wavefront. As the returned light level is high, we sample with the detector only a fraction of a speckle grain area instead of integrating the full area of the reflected light beam, to improve the contrast in the signal. By properly maintaining the interferometer in the quadrature working point, the out-of-plane surface displacements are proportional to the light power reaching the detector.\textsuperscript{26}

3. EXPERIMENT

3.1. Experimental system of TV holography

Figs. 3 and 4 show the TV holography system that we used in our experiments. The light source is a twin-cavity, frequency-doubled Nd:YAG pulsed laser with common injection seeder. The output green radiation is divided into an object beam, which illuminates the object surface after being expanded by a diverging lens, and a reference beam, which is guided by optical fibre and is slightly shifted off the optical axis to obtain a suitable spatial carrier in the interferograms. The light scattered by the object and that of the reference beam are combined with a beam combiner and imaged onto the photosensitive surface of the the CCD camera, which is thermoelectrically cooled and can record two images with full spatial resolution separated by 1\mu s. The Lamb wave generator is based on the prismatic coupling block method\textsuperscript{5} and comprises a programmable burst generator and a wedge that couples the ultrasound to an aluminium plate. Long bursts with 95 cycles have been used. The laser, the camera and the Lamb wave generator are synchronized by means of a computer and commercial and custom electronic devices.

3.2. Michelson speckle interferometer

We have built a very simple interferometer with “off-the-shelf” components without any refinements as could be polarizing optics, phase modulation or differential detection (Fig. 7). Given the bandwidth and sensitivity needed for our application, a 5mW He-Ne laser and a standard PIN detector perform well enough to detect the Lamb waves. More details of this instrument can be found in ref. 26.
Fig. 4. The TV holography system.

Amplitude (A.U.)

Time (μs)

Fig. 5. Tone-burst waveform recorded by the Michelson speckle interferometer at a distance of 230mm from the wedge front end, for the plate of 5mm thickness. Two Lamb modes, S0 and A0, of the same frequency are present.
4. RESULTS AND DISCUSSION

We have verified by Fourier transforming (Fig. 6) the waveforms acquired with the speckle point interferometer (Fig. 5) that, in all the experiments, the frequencies of all the existing modes were identical. This was also verified by the behavior of the nodal lines—lines of minimum contrast of the carrier—in the images of the displacement fields given by TVH: they were periodically located along the propagation direction at fixed distances from the generating transducer, regardless of the freezing instant.

Also, out-of-plane displacement maps were obtained with the TVH system for three plates of thickness 3, 4 and 5mm, respectively (Fig. 8). From these maps the carrier and beat wavelengths were measured and, following the procedure described in the section 2.1, the results of Table 1 were obtained. For lack of experimental data available, the nominal value of the Poisson’s ratio corresponding to the aluminum alloy utilized, \( \nu_{\text{nom}} = 0.33 \), was assumed. This leads to a value \( \lambda_{\text{nom}} = 1.99 \). The densities \( \rho \) were calculated from direct volume and mass measurements. The Young’s modulus \( E \) was calculated by the relation:

\[
E = 2\rho c_0^2 (1 + \nu)
\]

For comparison purposes, the nominal values of the alloy are included in the last row of the table.

<table>
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<th>( h ) (mm)</th>
<th>( N_s )</th>
<th>( N_A )</th>
<th>( \xi_{SA} )</th>
<th>( \xi_S )</th>
<th>( \Omega ) (for ( \lambda = 1.99 ))</th>
<th>( c_T ) (m/s)</th>
<th>( c_L ) (m/s)</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( E ) (GPa)</th>
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</table>

**NOMINAL VALUES**

| 3070  | 6100  | 2790  | 70.0  |

Table 1. Results.
Fig. 8. Out-of-plane instantaneous displacement maps of 1 MHz multimode Lamb waves of frequency 1.00 MHz propagating in aluminum plates of thicknesses: a) 2.98 mm, b) 4.03 mm, c) 5.00 mm. The real size of the field of view is 270 mm x 134 mm.
5. CONCLUSIONS

Lamb wave instantaneous displacement fields with intermode beating effects have been measured by pulsed TV holography. A theoretical analysis that allows to relate the carrier and beat wavelengths measured in the images to the elastic constants of the material was presented.

The fact that the frequencies of all the generated Lamb modes are coincident with the excitation frequency seems to be a consequence of using long bursts (with 95 cycles), so practically steady-state harmonic response is achieved (although the Lamb waves are transient in the sense that they are captured “in flight” before any reflection in the plate edges take place).

To reach solid conclusions about the accuracy of the results obtained, it would be necessary to have more reliable data of the elastic constants of the utilized plates (for example, by measuring the bulk wave velocities $c_L$ and $c_T$ by other independent means). Taking the nominal values as a reference, there are not strong incompatibilities. However, the intrinsic accuracy of the method is not very high because of the difficulty to measure the low-frequency beat wavelength in the images. In fact, the largest deviations from the nominal values have occurred with the 5mm plate (Fig. 8c), probably due to its large value (only two cycles fit into the image size) and poor definition of the nodes.

In conclusion, the method gives a rough estimation (better than 10%) of the bulk wave velocities but a refinement of the experimental uncertainties would be necessary to make it a practical tool to determine elastic constants.

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