

Antonio Fernández, Ángel F. Doval, Guillermo H. Kaufmann, Abundio Dávila, Jesús Blanco-García, Carlos Pérez-López and José L. Fernández, "Measurement of transient out-of-plane displacement gradients in plates using double-pulsed subtraction TV shearography," Opt. Eng. 39(8) 2106–2113 (August 2000)

Copyright 2000 Society of Photo-Optical Instrumentation Engineers.

This paper was published in "Optical Engineering" and is made available as an electronic reprint with permission of SPIE. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper are prohibited.

<http://dx.doi.org/10.1117/1.1305260>

# Measurement of transient out-of-plane displacement gradients in plates using double-pulsed subtraction TV shearography

**Antonio Fernández**, MEMBER SPIE  
Universidad de Vigo  
Department of Engineering Design  
Escuela Técnica Superior de Ingenieros  
Industriales  
Campus Universitario Lagoas-Marcosende  
E-36200 Vigo, Spain  
E-mail: antfdez@uvigo.es

**Ángel F. Doval**  
Universidad de Vigo  
Department of Applied Physics  
Escuela Técnica Superior de Ingenieros  
Industriales  
Campus Universitario Lagoas-Marcosende  
E-36200 Vigo, Spain

**Guillermo H. Kaufmann**, MEMBER SPIE  
Consejo Nacional de Investigaciones  
Científicas y Técnicas  
y Universidad Nacional de Rosario  
Instituto de Física de Rosario  
Bv. 27 de Febrero 210 bis  
2000 Rosario, Argentina

**Abundio Dávila**  
Centro de Investigaciones en Óptica  
Apartado Postal 1-948  
37000 León-Gto, Mexico

**Jesús Blanco-García**  
Universidad de Vigo  
Department of Applied Physics  
Escuela Unversitaria de Ingeniería Técnica  
Industrial  
Torrecedeira 86  
E-36208 Vigo, Spain

**Carlos Pérez-López**  
Centro de Investigaciones en Óptica  
Apartado Postal 1-948  
37000 León-Gto, Mexico

**José L. Fernández**  
Universidad de Vigo  
Department of Applied Physics  
Escuela Técnica Superior de Ingenieros  
Industriales  
Campus Universitario Lagoas-Marcosende  
E-36200 Vigo, Spain

**Abstract.** We report a technique for the measurement of transient out-of-plane displacement gradients in plane objects by double-pulsed subtraction TV shearography. The fringe patterns are automatically and quantitatively analyzed by the Fourier transform method. A novel optical setup based on the separation and further recombination of illumination beams is demonstrated for the generation of carrier fringes. The principle of the proposed technique is theoretically described, and its immunity to environmental disturbances is discussed. Experimental results obtained with a metallic plate excited by the impact of a piezoelectric transducer are presented. © 2000 Society of Photo-Optical Instrumentation Engineers. [S0091-3286(00)02908-1]

Subject terms: metrology; speckle interferometry; shearography; shock.

Paper 990116 received Mar. 18, 1999; revised manuscript received Nov. 10, 1999; accepted for publication Feb. 16, 2000.

## 1 Introduction

TV shearography (TVS)—or electronic speckle pattern shearing interferometry (ESPSI), as it is also called—is a nondestructive, whole-field technique that allows the mea-

surement of spatial derivatives of displacements. Early research on shearing techniques used moiré fringes resulting from the superposition of two fringe patterns obtained by holographic interferometry.<sup>1</sup> Photographic film was later

replaced by electronic devices, which avoid the somewhat expensive and time-consuming development process.<sup>2</sup> Gradient of displacement can also be obtained by the closely related technique of TV holography (TVH) through image processing.<sup>3</sup> However, in applications that involve the measurement of spatial derivatives of displacement (e.g., strain analysis and detection of local defects in various materials), TVS outstrips TVH in several respects. First, the calculation of spatial derivatives is a time-consuming operation, while TVS yields the slope of displacement directly. Second, due to its quasi-common-path design, TVS is less sensitive than TVH to the influence of environmental disturbances, e.g., air turbulence, external vibration, and rigid-body motion. Moreover, the requirement of a light source with large coherence length may be relaxed. And third, the ability to change the sensitivity of the interferometer by adjusting the amount of shear broadens the measurement range of TVS.

The use of pulsed lasers in TVS relaxes even more the stability requirements for the experimental setup and makes possible the analysis of high-speed transient events. Nevertheless, only a few papers reporting on pulsed TVS have appeared. Spooen et al.<sup>4</sup> originally demonstrated the application of a double-pulsed laser to electronic speckle shear interferometry. Shear is introduced in the speckle interferograms by slightly tilting one of the mirrors in a Michelson shear interferometer,<sup>5</sup> and correlation fringe patterns are formed by double-pulse subtraction,<sup>6</sup> a technique previously proposed for TVH. Emphasis was given to the study of fringe visibility rather than to the implementation of a phase evaluation method, and hence measurements remain qualitative. The first quantitative measurements of spatial derivatives of displacement using double-pulsed TVS have been carried out by Pedrini et al.<sup>7</sup> They use a Mach-Zehnder interferometer after the imaging lens in order to record the interference between two sheared images on a CCD. Lateral shear between these two images is adjusted by shifting one mirror in the setup, and the spatial carrier is introduced into the speckle shear interferograms by tilting one mirror (or one beamsplitter). The interference phase is evaluated using either the Fourier transform<sup>8</sup> or the spatial-carrier phase-shifting<sup>9</sup> method. The optical phase change due to the object's deformation is obtained as the difference between phase distributions calculated from two different speckle shear interferograms recorded before and after deformation, respectively. More recently, Dávila et al.<sup>10</sup> have applied pulsed TVS to quantitatively measure the slope of transient displacements following a different approach. The spatial carrier is introduced into the correlation fringe patterns rather than into the speckle shear interferograms, by translating manually the diverging lens that expands the illumination beam along its optical axis. The phase is then evaluated by the spatial synchronous detection method.<sup>11</sup> The technique has been experimentally demonstrated in laboratory conditions. Unfortunately, the long time required for the lens translation (several seconds) negates the advantages of pulsed TVS and prevents its application in industrial environments.

Bonding the diverging lens to a piezoelectric translator significantly improves the immunity of the system to environmental disturbances. This solution has been adopted in a double-pulsed addition TVH system for harmonic vibration

measurement.<sup>12</sup> However, it has been found that the use of a piezo-mounted lens to generate carrier fringes still imposes a lower limit to the minimum separation between laser pulses because of the response time of the piezoelectric element.

In this paper, we report a novel technique for the measurement of transient out-of-plane displacement gradients using double-pulsed subtraction TVS and the Fourier transform method (FTM). We have developed a new optical setup without moving devices to introduce the spatial carrier into the fringe patterns by changing the sensitivity vector between laser pulses. In contrast to the technique described in Ref. 10, the only lower limit to the minimum time separation between laser pulses is imposed by the charge-transfer period of the CCD, which can be as low as  $\sim 100$  ns in the latest models of the so called double-flash CCD cameras.<sup>13</sup>

## 2 Principle of the Technique

In this section we first present the general expression of a double-pulsed subtraction TVS fringe pattern. Later, we derive the phase-difference increment due to combined mechanical excitation and the proposed technique for spatial-carrier generation. Finally, we calculate the gradient of out-of-plane displacement by the FTM. Throughout this section we use certain approximations that allow phase-difference increment calculations to be reduced to simpler mathematical manipulations.

### 2.1 Fringe Formation in Double-Pulsed Subtraction TVS

In double-pulsed subtraction TVS the CCD camera records two speckle shear interferograms in separated video fields.<sup>6</sup> The first laser pulse is fired with the object at rest, and the corresponding intensity distribution may be expressed as

$$I_1(\mathbf{x}) = I_m(\mathbf{x})[1 + V(\mathbf{x}) \cos \Delta\psi_1(\mathbf{x})], \quad (1)$$

where  $I_m(\mathbf{x})$  and  $V(\mathbf{x})$  are the mean intensity and the visibility of the speckle shear interferogram at a point  $\mathbf{x} = (x, y, L)$  of the object's surface, respectively. (It should be emphasized that only plane objects are considered in this paper.) Some time later (typically tens of microseconds) the object is stressed and the laser emits a second pulse properly timed with respect to the mechanical excitation. The resulting intensity distribution is given by

$$I_2(\mathbf{x}) = I_m(\mathbf{x})[1 + V(\mathbf{x}) \cos \Delta\psi_2(\mathbf{x})]. \quad (2)$$

Here the functions  $\Delta\psi_1(\mathbf{x})$  and  $\Delta\psi_2(\mathbf{x})$  give the differences between the phases of the interfering speckle patterns for the first and the second speckle shear interferograms, respectively.

Subtraction of the intensity distributions (1) and (2) once they have been digitized, and subsequent full wave rectification, yields a speckled, high-visibility correlation fringe pattern

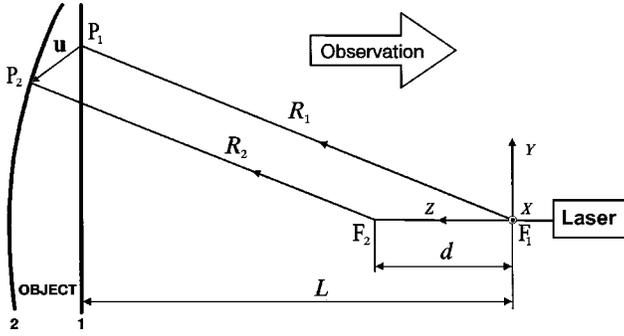


Fig. 1 Geometry of the optical-path separation approach.

$$I(\mathbf{x}) = |gI_2(\mathbf{x}) - gI_1(\mathbf{x})|$$

$$= 2gI_m(\mathbf{x})V(\mathbf{x}) \left| \sin \frac{\Delta\psi_1(\mathbf{x}) + \Delta\psi_2(\mathbf{x})}{2} \right| \left| \sin \frac{\Delta\phi(\mathbf{x})}{2} \right|, \quad (3)$$

where  $g = g(\lambda)$  is the spectral sensitivity of the camera for the laser wavelength  $\lambda$  and  $\Delta\phi(\mathbf{x})$  is the phase-difference increment

$$\Delta\phi(\mathbf{x}) = \Delta\psi_2(\mathbf{x}) - \Delta\psi_1(\mathbf{x}). \quad (4)$$

It should be noted that the modulus operation in Eq. (3) introduces harmonic distortion in the fringe pattern, which can give rise to significant errors in the recovered phase. For this reason we use quadratic detection rather than full-wave rectification for fringe analysis purposes:

$$i(\mathbf{x}) = I^2(\mathbf{x}) = i_m(\mathbf{x})[1 - \cos \Delta\phi(\mathbf{x})], \quad (5)$$

where the local mean intensity of the fringe pattern is given by

$$i_m(\mathbf{x}) = 2g^2I_m^2(\mathbf{x})V^2(\mathbf{x}) \sin^2 \frac{\Delta\psi_1(\mathbf{x}) + \Delta\psi_2(\mathbf{x})}{2}. \quad (6)$$

## 2.2 Phase Difference in Double-Pulsed Subtraction TVS

According to the geometry depicted in Fig. 1, the optical phase of one of the interfering speckle patterns produced by the first laser pulse may be expressed as

$$\psi_1(\mathbf{x}) = \psi_i + \psi_{F_1} + \frac{2\pi}{\lambda}R_1(\mathbf{x}) + \psi_s(\mathbf{x}) + \psi_{P_1}(\mathbf{x}), \quad (7)$$

with  $\psi_i$  the initial phase of the light source,  $\psi_{F_1}$  and  $\psi_{P_1}(\mathbf{x})$  the phase changes due to the propagation of light from the laser to the focus  $F_1$  and from a point  $P_1$  on the object surface to the image plane, respectively,  $\psi_s(\mathbf{x})$  a random-phase term due to the object surface roughness, and  $R_1(\mathbf{x})$  the distance from the focus  $F_1$  to the point  $P_1$  on the object.

In TVS, a point in the image plane receives contributions from two or more points on the object.<sup>14</sup> Let us consider a situation when the points  $\mathbf{x}$  and  $\mathbf{x} + \delta\mathbf{x}$  on the object

are imaged at the same point on the image plane, with  $\delta\mathbf{x} = (\delta x, \delta y)$  the object-plane shear. In that case, the phase difference of the first speckle shear interferogram is given by

$$\Delta\psi_1(\mathbf{x}) = \frac{2\pi}{\lambda}[R_1(\mathbf{x} + \delta\mathbf{x}) - R_1(\mathbf{x})] + \psi_s(\mathbf{x} + \delta\mathbf{x}) - \psi_s(\mathbf{x})$$

$$+ \psi_{P_1}(\mathbf{x} + \delta\mathbf{x}) - \psi_{P_1}(\mathbf{x}). \quad (8)$$

Making a first-order approximation

$$R_1(\mathbf{x} + \delta\mathbf{x}) - R_1(\mathbf{x}) \approx \frac{\partial R_1(\mathbf{x})}{\partial x} \delta x + \frac{\partial R_1(\mathbf{x})}{\partial y} \delta y$$

$$= \nabla R_1(\mathbf{x}) \cdot \delta\mathbf{x}, \quad (9)$$

Eq. (8) may be rewritten as follows:

$$\Delta\psi_1(\mathbf{x}) = \frac{2\pi}{\lambda} \nabla R_1(\mathbf{x}) \cdot \delta\mathbf{x} + \psi_s(\mathbf{x} + \delta\mathbf{x}) - \psi_s(\mathbf{x})$$

$$+ \nabla \psi_{P_1}(\mathbf{x}) \cdot \delta\mathbf{x}. \quad (10)$$

## 2.3 Spatial-Carrier Generation

The object surface undergoes a local displacement  $\mathbf{u}(\mathbf{x}) = [u(\mathbf{x}), v(\mathbf{x}), w(\mathbf{x})]$  because of mechanical excitation between exposures. Let us now suppose that the first and the second laser pulses propagate through separated paths, as shown in Fig. 1. Assuming that neither the object displacement nor the change in the illumination geometry introduces speckle decorrelation, the optical phase of the speckle pattern corresponding to the second laser pulse may then be expressed as

$$\psi_2(\mathbf{x}) = \psi_i + \psi_{F_2} + \frac{2\pi}{\lambda}R_2(\mathbf{x}) + \psi_s(\mathbf{x}) + \psi_{P_2}(\mathbf{x}), \quad (11)$$

where  $\psi_{F_2}$  and  $\psi_{P_2}(\mathbf{x})$  are the phase changes due to the propagation of light from the laser to the focus  $F_2$  and from a point  $P_2$  on the displaced object surface to the image plane, respectively, and  $R_2(\mathbf{x})$  is the distance from the focus  $F_2$  to the point  $P_2$  on the object.

In a first-order approximation, the phase difference of the second speckle shear interferogram is given by

$$\Delta\psi_2(\mathbf{x}) = \frac{2\pi}{\lambda} \nabla R_2(\mathbf{x}) \cdot \delta\mathbf{x} + \psi_s(\mathbf{x} + \delta\mathbf{x}) - \psi_s(\mathbf{x})$$

$$+ \nabla \psi_{P_2}(\mathbf{x}) \cdot \delta\mathbf{x}. \quad (12)$$

Provided that the in-plane components of the object displacement are negligible and that the object surface is placed perpendicular to the observation direction, we can approximate the difference of the phase changes due to the propagation of light from the points  $P_1$  and  $P_2$  to the image plane by

$$\psi_{P_2}(\mathbf{x}) - \psi_{P_1}(\mathbf{x}) \approx \frac{2\pi}{\lambda} w(\mathbf{x}), \quad (13)$$

and hence

$$[\nabla \psi_{P_2}(\mathbf{x}) - \nabla \psi_{P_1}(\mathbf{x})] \delta \mathbf{x} \approx \frac{2\pi}{\lambda} \nabla w(\mathbf{x}) \cdot \delta \mathbf{x}. \quad (14)$$

Thus, Eq. (4) may be rewritten in the form

$$\Delta \phi(\mathbf{x}) = \frac{2\pi}{\lambda} \nabla [R_2(\mathbf{x}) - R_1(\mathbf{x}) + w(\mathbf{x})] \cdot \delta \mathbf{x}. \quad (15)$$

Referring to Fig. 1, we can write

$$R_1(\mathbf{x}) = (x^2 + y^2 + L^2)^{1/2}, \quad (16)$$

$$R_2(\mathbf{x}) = \{x^2 + y^2 + [L + w(\mathbf{x}) - d]^2\}^{1/2}, \quad (17)$$

and introducing Eqs. (16) and (17) in Eq. (15) yields the rather long result

$$\begin{aligned} \Delta \phi(\mathbf{x}) = \frac{2\pi}{\lambda} \left( \left\{ \frac{x + [L + w(\mathbf{x}) - d] \frac{\partial w(\mathbf{x})}{\partial x}}{\sqrt{x^2 + y^2 + [L + w(\mathbf{x}) - d]^2}} \right. \right. \\ \left. \left. - \frac{x}{\sqrt{x^2 + y^2 + L^2}} + \frac{\partial w(\mathbf{x})}{\partial x} \right\} \delta x \right. \\ \left. + \left\{ \frac{y + [L + w(\mathbf{x}) - d] \frac{\partial w(\mathbf{x})}{\partial y}}{\sqrt{x^2 + y^2 + [L + w(\mathbf{x}) - d]^2}} - \frac{y}{\sqrt{x^2 + y^2 + L^2}} \right. \right. \\ \left. \left. + \frac{\partial w(\mathbf{x})}{\partial y} \right\} \delta y \right). \quad (18) \end{aligned}$$

Obviously, direct application of this general equation involves huge mathematical complexity. However, if we introduce certain approximations, calculations are reduced to simple linear operations. Our approximations will be based on two assumptions: (a) the distance  $L$  between the focus  $F_1$  and the object plane is much greater than any other distance that appear in Eq. (18), and (b) the distance  $d$  between the foci  $F_1$  and  $F_2$  is much greater than the out-of-plane displacement  $w(\mathbf{x})$ . These assumptions may be formalized as follows:

$$L \gg x, \quad L \gg y, \quad (19a)$$

$$L \gg d \gg w(\mathbf{x}). \quad (19b)$$

If the conditions above are satisfied, we can make a further approximation

$$\frac{ax + by}{(x^2 + y^2 + L^2)^{1/2}} \approx \frac{a}{L}x + \frac{b}{L}y, \quad (20)$$

where  $a$  and  $b$  are coefficients. Taking into account Eqs. (19) and (20) allows Eq. (18) to be rewritten as

$$\begin{aligned} \Delta \phi(\mathbf{x}) = 2\pi \frac{d}{\lambda L^2} x + \frac{4\pi}{\lambda} \frac{\partial w(\mathbf{x})}{\partial x} + 2\pi \frac{d}{\lambda L^2} y \\ + \frac{4\pi}{\lambda} \frac{\partial w(\mathbf{x})}{\partial y}. \quad (21) \end{aligned}$$

This result demonstrates that the phase difference increment in double-pulsed subtraction TVS with separated optical paths is the sum of two terms proportional to the spatial derivatives of displacement across horizontal and vertical coordinates as well as two terms proportional to the coordinates themselves. These last two terms may be considered as a spatial carrier whose frequency components (in units of fringes per image) are

$$f_{cx} = \frac{d}{\lambda L^2} X, \quad (22)$$

$$f_{cy} = \frac{d}{\lambda L^2} Y, \quad (23)$$

where  $X$  and  $Y$  are the horizontal and vertical fields of view, respectively.

## 2.4 Quantitative Determination of the Out-of-Plane Displacement Gradient

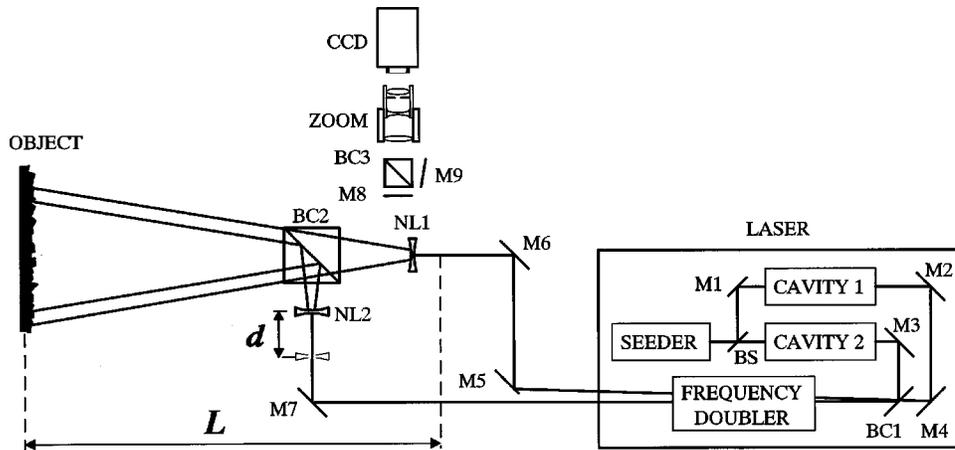
The phase-difference increment (21) can be extracted from the fringe pattern (5) by the FTM<sup>8</sup>

$$\Delta \phi(\mathbf{x}) = \text{unw} \left[ \tan^{-1} \left( \frac{\text{Im}[\mathcal{J}^{-1}\{\mathcal{J}\{i(\mathbf{x})\}W(\mathbf{f})\}]}{\text{Re}[\mathcal{J}^{-1}\{\mathcal{J}\{i(\mathbf{x})\}W(\mathbf{f})\}]} \right) \right], \quad (24)$$

where  $\mathcal{J}$  is the 2-D Fourier transform,  $W(\mathbf{f})$  is a suitable window in the Fourier plane, and unw denotes a generic phase-unwrapping operation.

Finally, to calculate the out-of-plane displacement gradient we proceed as follows. First, we introduce pure horizontal shear ( $\delta y = 0$ ) together with optical-path separation. The resulting fringe pattern shows a set of vertical, equally spaced carrier fringes modulated by the transient deformation. We denote the corresponding phase difference increment by  $\Delta \phi_x^d(\mathbf{x})$ . In order to isolate the term proportional to the displacement derivative, we subtract the carrier contribution from the recovered phase. This method is called subtraction of the phase of the undeformed carrier fringes (SPUCF).<sup>15</sup> The carrier contribution in the horizontal direction,  $\Delta \phi_x^c(\mathbf{x})$ , is evaluated from a fringe pattern obtained without mechanical excitation. Next, the interferometer is arranged to introduce pure vertical shear ( $\delta x = 0$ ), so that the resulting carrier fringes are now a set of horizontal, equally spaced fringes. The experiment is then repeated with and without mechanical excitation. We denote the corresponding phase difference increments by  $\Delta \phi_y^d(\mathbf{x})$  and  $\Delta \phi_y^c(\mathbf{x})$ , respectively. Gradient determination is straightforward from these definitions

$$\nabla w(\mathbf{x}) = \frac{\lambda}{4\pi} \left( \frac{\Delta \phi_x^d(\mathbf{x}) - \Delta \phi_x^c(\mathbf{x})}{\delta x}, \frac{\Delta \phi_y^d(\mathbf{x}) - \Delta \phi_y^c(\mathbf{x})}{\delta y} \right). \quad (25)$$



**Fig. 2** Schematic experimental setup used to generate carrier fringes in double-pulsed-subtraction TV shearography: BS, beamsplitter; BC1 to BC3, beam combiners; M1 to M9, mirrors; NL1 and NL2, negative lenses (their foci are shifted by a distance  $d$ ).

### 3 Optical Setup

Our experimental setup for spatial-carrier generation in double-pulsed TVS is shown in Fig. 2. The light source consists of two separate  $Q$ -switched Nd:YAG oscillators, commonly seeded by a diode-pumped Nd:YAG cw laser to obtain mutual coherence between their beams. The infrared outputs are combined at BC1 before passing through a frequency-doubling crystal that makes alignment tasks and signal detection safer and easier. Moreover, the sensitivity of the interferometer is doubled. Each cavity produces 12 mJ in 20-ns pulses at  $\lambda = 532$  nm at a rate of 25 Hz. Giving a small tilt to mirror M4, the green radiation produced by cavity 1 goes through a different path than light produced by cavity 2. Both beams are expanded through diverging lenses and then are made collinear at BC2. The lens NL2 is placed a distance  $d$  (exaggerated in the diagram for clarity) nearer to BC2 than NL1, and therefore the curvature radius of the illumination beam is greater for the first laser pulse than for the second, as required for spatial-carrier generation (Fig. 1). The lens NL2 is mounted on a translation stage that allows the distance  $d$  to be precisely adjusted. Either horizontal or vertical shear is introduced in the speckle interferograms by slightly tilting the mirror M9 in the Michelson arrangement placed in front of the imaging system.

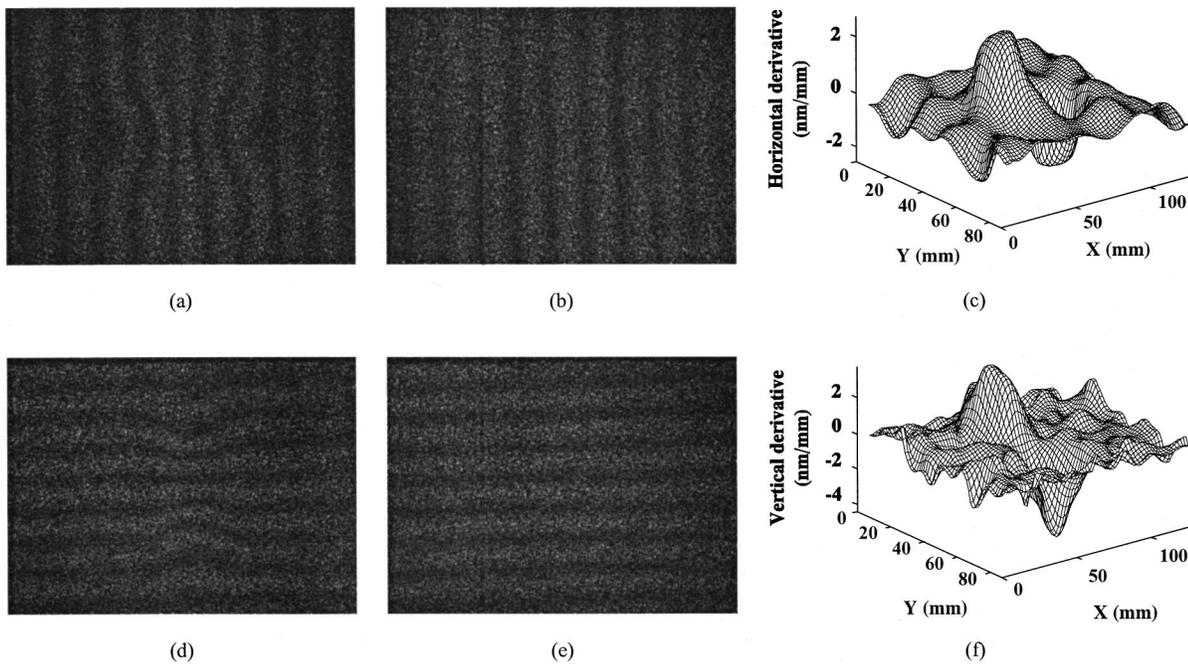
### 4 Experimental Results

We have demonstrated the optical setup schematically represented in Fig. 2 for the measurement of the out-of-plane displacement gradient of impact-induced transient bending waves in metallic plates. The operation of our double-pulsed TVS system is as follows. The laser continuously emits twin pulses at a rate of 25 Hz, properly timed with respect to the video signal. When the operator gives the order to start, the test object receives an impact from a piezoelectric transducer, and a video frame is digitized and stored as  $512 \times 512 \times 8$  bits in a framegrabber. The irradiance values recorded by the CCD during both the first and the second laser pulses, Eqs. (1) and (2), are contained in the even and the odd lines of that image, respectively. Intensity fluctuations between first and second laser pulses

are digitally compensated. Next, a frame processor subtracts even lines from the adjacent odd ones and rectifies the result, Eq. (5), yielding a set of equally spaced straight carrier fringes modulated by the spatial derivative of normal displacement. (The interested reader is directed to Refs. 16 and 17 for further details on the synchronism system.)

The specimen used in our experiment is an aluminum plate ( $300 \times 120 \times 3.5$  mm) clamped along its left and its right edges. An area of approximately 124 mm ( $X$ ) by 84 mm ( $Y$ ) was measured. The object was impact-excited at the central point of the back side of this area. The time separation between laser pulses was set to  $50 \mu\text{s}$ . The delay between the piezoelectric transducer driving signal and the firing of the second laser pulse was adjusted to  $20 \mu\text{s}$  by means of a programmable delay generator. We set the distances  $d$  and  $L$  to 6.35 mm and 2.09 m, respectively. The numerical values of horizontal and vertical object shear are  $\delta x = 32.5$  mm and  $\delta y = 37$  mm, respectively. One can calculate horizontal as well as vertical carrier frequencies using Eqs. (22) and (23). For the experimental conditions just mentioned, the theoretical values are  $f_{cx} = 11.01$  and  $f_{cy} = 8.49$ , expressed in fringes per image. These predictions are in good agreement with experimental results.

Figure 3(a) is an experimental fringe pattern obtained with pure horizontal shear, which shows vertical carrier fringes modulated by the transient deformation. Figure 3(b) is the fringe pattern resulting from the repetition of the experiment without mechanical excitation. Application of the FTM, Eq. (24), to the fringe patterns in Figs. 3(a) and 3(b) yields  $\Delta \phi_x^d(\mathbf{x})$  and  $\Delta \phi_x^c(\mathbf{x})$ , respectively (see Sec. 2.4). We used a filtering window  $W(\mathbf{f})$  that yields 1 for the points inside a circle and 0 otherwise. Following the approach of Takatsuji et al.,<sup>18</sup> we set the frequency coordinates of the center of this circular domain to the same values as the peak of the sidelobe of the Fourier spectrum, and its diameter to the maximum value that yields a phase map without strong discontinuities (15 pixels for the fringe patterns in Fig. 3). Wrapped phase maps were unwrapped using an algorithm based on a least-squares minimization technique that is solvable by the discrete cosine



**Fig. 3** Transient bending waves in a metal plate 25  $\mu$ s after mechanical excitation: (a) to (c), horizontal shear; (d) to (f), vertical shear; (b), (e), carrier fringes; (a),(d), corresponding double-pulsed-subtraction TV shearography fringe patterns, deformation plus carrier; (c), (f) three-dimensional plots of the spatial derivatives of the out-of-plane displacement.

transform.<sup>19</sup> In this approach, invalid pixels due to phase inconsistencies and regions with missing data are excluded from the unwrapping process through the assignment of zero-valued weights. In this way, the algorithm can smoothly interpolate the phase over pixels with bad data.

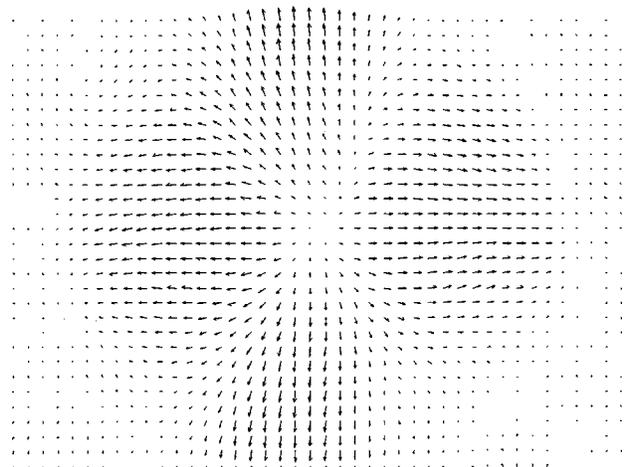
The weighted problem is solved using a preconditioned conjugate-gradient method, which gives guaranteed and faster convergence. The spatial derivative along the horizontal direction can be determined by subtracting  $\Delta\phi_x^d(\mathbf{x})$  from  $\Delta\phi_x^c(\mathbf{x})$  and scaling the result, Fig. 3(c). Following an analogous procedure, we evaluated the phase distributions  $\Delta\phi_y^d(\mathbf{x})$  and  $\Delta\phi_y^c(\mathbf{x})$  from their corresponding experimental fringe patterns obtained with pure vertical shear, Figs. 3(d) and 3(e), to determine the spatial derivative along the vertical direction, Fig. 3(f). Finally, Fig. 4 is a representation of the out-of-plane displacement gradient.

## 5 Discussion

We have developed an optical setup for spatial-carrier generation without moving parts. This makes our system highly immune to environmental disturbances, because the time separation between laser pulses can be as short as the transfer period of the CCD camera (see Sec. 1). However, the inherent immunity of our system to noise is strongly affected by the method employed for carrier removal. We have used SPUCF because it is the method that introduces the lowest errors in the phase distribution; nevertheless, the immunity to environmental disturbances is dramatically reduced due to the relatively long time elapsed between the recording of the two necessary fringe patterns (40 ms). This fact may be disregarded as long as the working conditions are controlled, as they are in a laboratory, but it is relevant for operation in industry. In that case, the phase must be

calculated from a single fringe pattern, and therefore SPUCF is inadequate. The solution is to use other carrier removal methods (e.g., translation of the sidelobe to the frequency origin, or least-squares fitting), that do not require a second fringe pattern for phase evaluation. The influence of the main existing carrier removal methods on the accuracy of the results and on the immunity to noise is discussed in more depth in Ref. 15.

Obviously, the arguments above are completely true only for the calculation of one component of the displacement gradient, either the horizontal or the vertical spatial derivative, because of the long time required (tens of sec-



**Fig. 4** 2-D representation of the transient out-of-plane displacement gradient in the central area of the tested specimen.

onds) to repeat the experiment with a different shear direction.

Finally, it is possible to obtain a sequence of quantitative measurements showing the temporal evolution of the measured magnitude by repeating the experiment with a variable time delay between the mechanical impact and the second laser pulse.

## 6 Conclusions

We have reported a new technique for the measurement of transient out-of-plane displacement derivatives by double-pulsed subtraction TV shearography. The introduction of carrier fringes by mismatching the distances from the diverging lenses to the beam combiner allows quantitative analysis of the fringe patterns using the Fourier transform method. The inherent immunity of our system to noise, which is a valuable feature for industrial application, is strongly affected by the method employed for carrier removal. We have shown experimental results with a metallic plate excited by impact to illustrate the performance of our approach.

## Acknowledgments

The authors thank for their support the following institutions: Xunta de Galicia (XUGA 32105B97), Comisión Interministerial de Ciencia y Tecnología (TAP97-0829-C03-01), and Universidad de Vigo (64502I801). Part of this work was presented at the SPIE's International Symposium on Optical Science, Engineering and Instrumentation.<sup>20</sup>

## References

1. P. Boone and R. Verbiest, "Application of hologram interferometry to plate deformation and translation measurements," *Opt. Acta* **16**, 555–567 (1969).
2. S. Nakadate, T. Yatagai, and H. Saito, "Digital speckle-pattern shearing interferometry," *Appl. Opt.* **19**, 4241–4246 (1980).
3. E. Vikhagen, "Nondestructive testing by use of TV holography and deformation phase gradient calculation," *Appl. Opt.* **29**, 137–144 (1990).
4. R. Sporeen, A. A. Dyrseth, and M. Vaz, "Electronic shear interferometry: application of a (double-) pulsed laser," *Appl. Opt.* **32**, 4719–4727 (1993).
5. P. K. Rastogi, "Techniques of displacement and deformation measurements in speckle metrology," in *Speckle Metrology*, R. S. Sirohi, Ed., pp. 41–98, Marcel Dekker, New York (1993).
6. R. Sporeen, "Double-pulse subtraction TV holography," *Opt. Eng. (Bellingham)* **31**, 1000–1007 (1992).
7. G. Pedrini, Y.-L. Zou, and H. J. Tiziani, "Quantitative evaluation of digital shearing interferogram using the spatial carrier method," *Pure Appl. Opt.* **5**, 313–321 (1996).
8. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry," *J. Opt. Soc. Am.* **72**, 156–160 (1981).
9. M. Kujawinska, "Spatial phase measurement methods," in *Interferogram Analysis*, D. W. Robinson and G. T. Reid, Eds., pp. 141–193, Institute of Physics Press, Bristol (1993).
10. A. Dávila, G. H. Kaufmann, and C. Pérez-López, "Transient deformation analysis using a carrier method of pulsed electronic speckle-shearing pattern interferometry," *Appl. Opt.* **37**, 4116–4122 (1998).
11. K. H. Womack, "Interferometric phase measurement using spatial synchronous detection," *Opt. Eng. (Bellingham)* **23**, 391–395 (1984).
12. A. J. Moore and C. Pérez-López, "Fringe carrier methods in double-pulsed addition ESPI," *Opt. Commun.* **141**, 203–212 (1997).
13. W. Steinchen, L. X. Yang, G. Kupfer, P. Mäckel, and F. Vössing, "Developmental steps to double pulse shearography," in *Laser Interferometry IX: Techniques and Analysis*, M. Kujawinska, G. M. Brown, and M. Takeda, Eds., *Proc. SPIE* **3478**, 344–351 (1998).
14. R. S. Sirohi, "Speckle methods in experimental mechanics," in *Speckle Metrology*, R. S. Sirohi, Ed., pp. 99–156, Marcel Dekker, New York (1993).
15. A. Fernández, G. H. Kaufmann, A. F. Doval, J. Blanco-García, and J. L. Fernández, "Comparison of carrier removal methods in the analysis of TV holography fringes by the Fourier transform method," *Opt. Eng. (Bellingham)* **37**, 2899–2905 (1998).
16. A. Fernández, A. J. Moore, C. Pérez-López, A. F. Doval, and J. Blanco-García, "Study of transient deformations with pulsed TV holography: application to crack detection," *Appl. Opt.* **36**, 2058–2065 (1997).
17. A. Fernández, J. Blanco-García, A. F. Doval, J. Bugarín, B. V. Dorrio, C. López, J. M. Alén, M. Pérez-Amor, and J. L. Fernández, "Transient deformation measurement by double-pulsed-subtraction TV holography and the Fourier transform method," *Appl. Opt.* **37**, 3440–3446 (1998).
18. T. Takatsuji, B. F. Oreb, D. I. Farrant, and J. R. Tyrer, "Simultaneous measurement of three orthogonal components of displacement by electronic speckle-pattern interferometry and the Fourier transform method," *Appl. Opt.* **36**, 1438–1445 (1997).
19. G. H. Kaufmann, G. E. Galizzi, and P. D. Ruiz, "Evaluation of a preconditioned conjugate-gradient algorithm for weighted least-squares unwrapping of digital speckle-pattern interferometry phase maps," *Appl. Opt.* **34**, 3076–3084 (1998).
20. A. Fernández, A. F. Doval, A. Dávila, J. Blanco-García, C. Pérez-López, and J. L. Fernández, "Double-pulsed carrier speckle-shearing pattern interferometry for transient deformation analysis," in *Laser Interferometry IX: Techniques and Analysis*, M. Kujawinska, G. M. Brown, and M. Takeda, Eds., *Proc. SPIE* **3478**, 352–358 (1998).



**Antonio Fernández** received his Industrial Engineer and his Doctor of Industrial Engineering degrees in 1993 and 1998, respectively, from the University of Vigo, Spain. He is currently an associate lecturer in the Department of Engineering Design of the University of Vigo. His main research interests are speckle metrology, colorimetry, fringe pattern analysis and digital image processing. He is a member of SPIE.



**Ángel F. Doval** is a lecturer in applied physics at the University of Vigo, Spain. He received the degrees of Industrial Engineer from the University of Santiago de Compostela in 1990 and Doctor in Industrial Engineering from the University of Vigo in 1997. His main research interests are TV holography (ESPI) and related techniques, fringe pattern analysis, and the use of fiber optics and laser diodes in interferometric systems.



**Guillermo H. Kaufmann** received his DSc degree in physics in 1978 from the Universidad de Buenos Aires, Argentina. He is currently a professor in the Physics Department of the Universidad Nacional de Rosario and a research scientist of the Consejo Nacional de Investigaciones Científicas y Técnicas of Argentina. He is also the head of the Division of Experimental and Applied Physics at the Instituto de Física Rosario, where he leads a group working in optical metrology. He has worked as a visiting researcher at the National Physical Laboratory (UK), the University of Michigan, the Swiss Federal Institute of Technology at Lausanne, the University of Cambridge, the Mechanical Engineering Laboratory in Japan, and Loughborough University (UK). Professor Kaufmann has authored three book chapters and more than 70 scientific papers published in refereed journals and conference proceedings. His major research interests include the development of coherent optics techniques for strain analysis and nondestructive testing, speckle metrology, phase-shifting interferometry, fringe analysis, and digital image processing. He is a member of OSA, SPIE, and IEEE.

**Abundio Dávila** received his BS in physics from the Universidad Autónoma de Nuevo León, México, in 1983 and his MSc degree from the Centro de Investigación Científica y de Educación Superior de Ensenada in 1986. He joined the Centro de Investigaciones en

Óptica (CIO) in 1986, obtaining his PhD from Loughborough University in 1996. Now his research is continuing in the area of speckle metrology at CIO. His research interests lie in the areas of transient event detection using electronic speckle pattern interferometry, speckle noise reduction, fringe analysis, and digital image processing.



**Jesús Blanco-García** received his doctoral degree in physics in 1992, with a thesis on holographic interferometry, from the University of Santiago de Compostela. He currently teaches general physics at the University of Vigo. His main research subjects are TV holography, holographic interferometry, moiré methods, interferometers, and fringe analysis.

**Carlos Pérez-López** received his BSc in electronic engineering from the Instituto Tecnológico y de Estudios Superiores de Monter-

rey, México, and his MSc in optics from the Centro de Investigaciones en Óptica in 1985. He is currently a PhD student at the Centro de Investigaciones en Óptica.



**José L. Fernández** received his diploma in mechanical engineering from Universidad Politécnica de Madrid in 1984 and since has been with the Optical Metrology group of the Industrial Engineering School of Vigo, which until 1989 was affiliated with the Universidad de Santiago de Compostela, where he received his PhD degree in engineering in 1988. He is currently a senior lecturer at the Industrial Engineering School of Vigo, now part of the Universidad de Vigo. His interests include dimensional measurements and non-destructive inspection by optical means and, more specifically, cw and pulsed TV holography, interferometry, and moiré techniques.