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Spatiotemporal 3D Fourier transform evaluation of speckle interferogram sequences in double-pulsed TV holography

C. Trillo*, A. F. Doval
Universidade de Vigo, Departamento de Física Aplicada, E. T. S. E. Industriais
Campus Universitario 36310 Vigo (Spain)

ABSTRACT

Phase evaluation based on the spatial Fourier transform of speckle interferograms usually assumes that the side lobes corresponding to the interferential terms in the Fourier spectrum of each interferogram do not overlap the terms related to the intensity of the object and reference beams. If this is not the case, a part the autocorrelation of the object beam is taken along with the selected interference term in the subsequent filtering process and induces an error in the resultant phase map. We present a technique for the acquisition and processing of speckle interferogram sequences that separates the interference lobes from the other spectral terms, even if the aforementioned assumption does not apply. A digital camera records a sequence of temporally phase-shifted interferograms with spatial carrier, and a 3D Fourier transform is applied to this set of spatiotemporal data. In the resultant three-dimensional spectrum, the temporal carrier shifts the central temporal frequency of the interferential terms to high frequencies. This minimizes their overlap with the autocorrelation of the object and reference beams, which are essentially located at low temporal frequencies. The spatial carrier prevents a possible overlap of broadband interferential terms along the temporal frequency axis. A filter that rejects the low temporal frequencies is applied to one of the lobes, and a sequence of complex-valued maps containing the optical phases of the interferograms is obtained. The measurement of surface acoustic waves propagating in a metallic plate, obtained with a double-pulsed TV holography setup, is presented to illustrate the method.

Keywords: Digital holography, three-dimensional image processing

1. INTRODUCTION

The two-dimensional Fourier transform method1 (2DFT) —a widely used technique for retrieving the phase of interferograms with spatial carrier— yields the optical phase of an interferogram through the calculation of a direct 2D Fourier transform, the selection of one interferential term of the spectrum by means of a filter, and an inverse 2D Fourier transform. The usual assumption is that the Fourier spectrum of the interferograms has a trimodal distribution, i. e., the interferential terms are completely separated from the low-frequency terms related to the intensity of the object and reference beams. When this assumption does not apply, the interferential terms partially overlap the spectrum of the object beam, as shown in Figure 1(b), which thus induces an error in the retrieved optical phase. The four spectral terms can be distinguished in this image: the narrow bright region around the origin is the term related to the intensity of the reference beam. The faint broad halo centered in the image is related to the intensity of the object beam. The interferential terms are focused images of the aperture, symmetrically shifted from the origin by the spatial carrier; this is the usual case in image-plane digital holography when the reference beam is made to diverge from a point on the aperture plane. Whilst total spectral separation could still be achieved by reducing the size of the aperture, it would be at the cost of reducing the amount of light reaching the sensor and decreasing the spatial resolution. Changing the shape of the aperture (e.g., a narrow rectangular window) is not an option when working with commercial camera objectives. We propose herein a method to separate the spectral terms that requires the recording of a sequence of interferograms with spatial carrier, the introduction of a temporal carrier by modulating the reference beam between successive acquisitions, and a 3D Fourier transform of this set of spatiotemporal data. In the resultant 3D spectrum, the spatial and temporal carriers separate the interferential lobes from one another and from the other two terms. A sequence of complex-valued maps containing the optical phases of the interferograms is obtained after the application of a 3D filter and an inverse 3D Fourier transform.

*mctrillo@uvigo.es, phone +34 986 812216
This procedure, designated as 3DFT henceforth, follows the same rationale that the authors used some years ago to develop a technique\textsuperscript{3} to obtain the mechanical complex amplitude of ultrasonic surface acoustic waves from a sequence of displacement maps, and was also applied some time later\textsuperscript{4} in the field of fringe projection profilometry.

2. THEORETICAL BACKGROUND

A speckle interferogram with spatial carrier, recorded with single-pulse illumination, for a given instant \( t_n \) can be written

\[
I_n(x) = I_{o,n} + I_{r,n} + 2\sqrt{I_{o,n}I_{r,n}} \cos(\phi_n + 2\pi f_{cx} \cdot x) \tag{1}
\]

where \( x=(x_1,x_2) \) is the position on the image plane, \( I_n(x) \) is the intensity of the \( n \)-th interferogram, \( I_{o,n}=I_{o}(x) \) and \( I_{r,n}=I_{r}(x) \) are the intensities of the object and reference beams respectively. The phase term \( \phi_n(x,t_n) \) stands for \( \phi_n = \psi_{p,n} + \phi_{o,n} - \phi_{r,n} \), where \( \psi_{p,n}=\psi_{p}(x) \) is the random phase due to the speckle, \( \phi_{o,n}=\phi_{o}(x,t_n) \) is the object phase related to the displacements of the object and \( \phi_{r,n}=\phi_{r}(x,t_n) \) is the reference phase. The term \( 2\pi f_{cx} \cdot x \), with \( f_{cx}=(f_{cx1},f_{cx2}) \), is the spatial carrier. Fringe visibility corresponding to perfect coherence has been assumed, as it is a realistic condition in a well-adjusted experimental set-up. \( N \) interferograms \( I_n \) with \( n=0,\ldots,N-1 \) are acquired, and an additional phase \( \alpha_n \) is introduced in each interferogram. The increment of \( \alpha_n \) from one interferogram to the next is the phase step \( \alpha \) and is the same for all the interferograms, i. e. \( \alpha_{n+1}-\alpha_n=\alpha \). The additional phase \( \alpha_n \) can be seen as a temporal carrier of the form \( 2\pi f_{ct_n} \), with a frequency

\[
f_{ct} = \frac{\alpha}{2\pi (t_{n+1}-t_n)} = \frac{\alpha}{2\pi \Delta t} \tag{2}
\]

The sequence of interferograms with spatial and temporal carriers can then be expressed as

\[
I(x,t) = I_o + I_t + 2\sqrt{I_o I_t} \cos(\phi + 2\pi f_{cx} \cdot x + 2\pi f_{ct} t) \tag{3}
\]

where \( I_o \) and \( I_t \) are now functions of \( x \) and \( t \). This last equation can be rewritten as
Figure 2. Main elements of the experimental set-up

\[ I = I_o + I_r + 2\sqrt{I_o I_r} \left\{ \frac{1}{2} \exp[i(\phi + 2\pi f_{c_x} \cdot x + 2\pi f_{c_t} t)] + C^* \right\} \]  

where \( C^* \) is the complex conjugate of the first term in the braces. The Fourier transform of Eq. (4) is

\[ F(I) = F(I_o) + F(I_r) + F(\sqrt{I_o I_r}) \left\{ \left[ F[\exp(i\phi)] \ast \delta(f_x - f_{c_x}, f_t - f_{c_t}) + F(C^*) \right\} \]  

where \( F \) designates the Fourier transform operator. The first two terms, related to the intensities of the object and reference beams, respectively, change slowly with time, so they are essentially located at low temporal frequencies. The remaining terms correspond to the interferential lobes, are shifted by the spatial and temporal carriers to frequencies \((f_{c1}, f_{c2}, f_{c3})\) and \((-f_{c1}, -f_{c2}, -f_{c3})\) (see Figure 1(d)). If the frequencies of the carriers are high enough, the lobes are separated from one another and from the other two terms. A 3D bandpass filter, in the general case, selects one of the side lobes (Figures 1(d), 1(e)) and the inverse Fourier transform of the filtered data gives a set of complex-valued phase maps

\[ I'(x,t) = \sqrt{I_o I_r} \exp(i\phi) \exp[i(2\pi f_{c_x} \cdot x + 2\pi f_{c_t} t)] \]  

whose arguments contain the optical phases \( \Phi_n(x,t) = \phi_n+2\pi f_{c_x} x + 2\pi f_{c_t} t \) of the sequence of interferograms (Figure 1(f)).

It would seem that the goal of spectral separation could be achieved with a temporal carrier alone and a pixel-wise temporal Fourier transform.\(^6\) However, if the spectral content of the interferential terms spreads along the temporal frequency axis —due to the dynamics of the object, a modulation of the intensity of the object or reference beams or an imperfect phase shifting— the conjugate terms could partially overlap, leading to an error in the retrieved phase. This situation can be avoided with the introduction of a spatial carrier in the data.

3. EXPERIMENTAL

3.1 Setup

The main elements of the experimental set-up are shown in Figure 2. The light source was a frequency-doubled, injection-seeded, Nd:YAG pulsed laser with two independent cavities running at 25 Hz (Spectron SL404 T), which delivered pairs of pulses with \( \lambda = 532 \) nm. A Mach-Zehnder interferometer with out-of-plane sensitivity split the laser output into a reference beam, which was coupled to a monomode optical fiber, and an object beam, which was expanded to illuminate the object. A photographic objective was used to form an image of the sample on the sensor of a thermoelectrically cooled CCD camera (PCO Double Shutter). The object was an aluminium slab of dimensions 400×130×30 mm\(^3\). It was insonified with short bursts of ultrasonic surface acoustic waves that induced a nanometric displacement of the observed surface. The interferometer was sensitive to the out-of-plane component of this
displacement. The waves were generated with a piezoelectric transducer, excited with high voltage bursts of central frequency 1.000 MHz, and were coupled to the sample with a plastic wedge with the appropriate angle. The camera recorded the interference of the reference beam and the light backscattered from the object in a sequence of \( N = 64 \) pairs of interferograms \((I^r_n, I^l_n)\), with 12 bits per pixel and dimensions 1280×1024 pixels. \( I^r_n \) and \( I^l_n \) denote individual interferograms located at position \( n \) in the sequence, with \( n = 0, ..., N-1 \). Each interferogram \( I^r_n \) corresponded to the reference state and was recorded with the first pulse of one pair of laser pulses. The second pulse of that same pair was used to record the interferogram \( I^l_n \), which captured a second state of the surface. The interval between the two interferograms of each pair \((I^r_n, I^l_n)\) was 1.5 μs. A spatial carrier was introduced in each interferogram by tilting the output of the reference beam slightly off the optical axis. A phase stepper, constructed by wrapping the optical fiber around a cylindrical piezoelectric transducer, was used to modulate the phase of the reference beam and introduce a phase step \( \alpha = \pi/2 \) between consecutive pairs of interferograms. The recording speed was limited by the transfer rate from the camera to the computer to approximately 3 pairs of interferograms per second. The total recording time was approximately 21 s. The emission of the laser pulses, the excitation of the ultrasonic wave and the trigger of the camera were synchronized with a delay generator (DG535, Stanford Research Systems) and ad hoc electronics. Due to instabilities in the laser source (see section 3.2) we tried to keep the acquisition rate as high as possible, so we eliminated non-essential operations between the acquisition of consecutive pairs of interferograms. For that reason, the time interval between the excitation of the ultrasonic wave and the laser pulses was kept constant and the wavetrain was thus captured at the same position on the surface in all the images. All the calculations and image processing were carried out in a laptop computer equipped with an Intel Core2 Duo CPU operating at 1.83 GHz and 3GB of RAM memory.

3.2 3D Fourier transform processing

The obtained sequence of interferograms is actually a discrete 3D set of experimental data of \( P \times Q \times N \) sampled points

\[
 I(x_{1p}, x_{2q}, t_n) = I(x_{10} + p\Delta x_1, x_{20} + q\Delta x_2, t_0 + n\Delta t)
\]

with \( p = 0, ..., P-1 \), \( q = 0, ..., Q-1 \), and \( n = 0, ..., N-1 \). \( \Delta x_1 \) and \( \Delta x_2 \) are the spatial sampling distances in the horizontal and vertical directions respectively, and \( \Delta t \) is the temporal sampling interval. The resultant 3D spectrum is thus also discrete, and can be viewed as a sequence of \( N \) planes corresponding to \( N+1 \) discrete temporal frequencies \( f_n \), where each plane contains a set of discrete spatial frequencies \( f_p, f_q \). The equations of these discrete frequencies are given by

\[
f_{p'} = \frac{p}{P\Delta x_1}, \quad f_{q'} = \frac{q}{Q\Delta x_2}, \quad f_{n'} = \frac{n'}{N\Delta t}
\]

with \( p' = -P/2, ..., P/2 \), \( q' = -Q/2, ..., Q/2 \) and \( n' = -N/2, ..., N/2 \). The extreme values of \( n' \), corresponding to the Nyquist critical temporal frequencies \( \pm f_N = \pm 1/(2\Delta t) \), lie on the same plane. Taking this into account, \( n' = -(N/2)-1, ..., N/2 \) is used as the index along the temporal frequency axis in the following figures.

The central temporal frequency of the interference lobes appears at two symmetric planes \( \pm n' \) determined by the temporal carrier \( 2\pi f_{ct} \). To calculate the value of \( n' \) in our experiments, we use Eq. (2) with \( \alpha = \pi/2 \), which yields a temporal carrier \( f_{ct} = 1/(4\Delta t) \), and from Eq. (9), \( n' \) is obtained as

\[
n' = \pm N\Delta t f_{ct} = \pm \frac{N}{4}
\]

To verify this point, the 3D Fourier transform of a sequence of 8 consecutive interferograms \( I^r_n \), with \( n = 15, ..., 22 \), was calculated. The resultant 3D spectrum consists of eight complex-valued planes whose moduli are shown in Figure 3(a). Though the spectral content of each lobe is spread along the temporal frequency axis, it is strongest at plane \( n' = \pm 8/4 = \pm 2 \), in agreement with Eq. (10). This qualitative result was verified by computing the average modulus of each map in the region bounded by the dotted rectangle of map \( n' = 2 \) in Figure 3(a), and by plotting the resultant values versus \( n' \) (see Figure 3(b)). The relatively high value at \( n' = 0 \) comes essentially from the spectrum of the object beam that enters the region. These eight pairs of interferograms \((I^r_n, I^l_n)\), with \( n = 15, ..., 22 \), were used to obtain the results presented in the following sections.

The spread of the interferential lobes has its cause in a fluctuation of the phase of the laser pulses, in addition to the relatively low acquisition rate available with our camera. This fluctuation added random phase jumps to the \( \pi/2 \) phase step, or, in other words, it broadened the spectrum of the temporal carrier, which is ideally a \( \delta \) function (Eq. (5)).
We observe that the spatial carrier avoids the overlap of the interferential terms along the $f_c$ axis, as mentioned at the end of section 2.

The 3DFT method sketched in Figure 4 was first applied to interferograms $I^a_n$ and then to interferograms $I^b_n$. The dimensions of the 3D filter, shown in Figure 4(b), were 231×231 pixels in $f_p'$ and $f_q'$; in $f_n'$ it was a band-stop filter that removed the plane $n'=0$, where the spectrum of the object beam was confined. Two sequences of 8 complex-valued maps $(I^a_n, I^b_n)$, containing the optical phases $\Phi^{a}_{n} = \text{arg}(I^a_n)$ and $\Phi^{b}_{n} = \text{arg}(I^b_n)$ of the interferograms, were thus obtained (see Figure 4(c)). The calculations were carried out with an implementation of the fast Fourier transform algorithm\(^8\) which required that all the dimensions were integer powers of two; hence, only a region of interest of 512×512 pixel in each interferogram was considered.

### 3.3 Calculation of optical phase-change maps $\Delta \Phi_n$

The quantity of interest is usually the optical phase-change $\Delta \Phi$ between a reference state and a second excited state. In our case, the optical phase change for a given $n$ is $\Delta \Phi_n = \Phi^b_n - \Phi^a_n$, where $\Phi^b_n$ and $\Phi^a_n$ are the optical phases of the excited and reference states respectively. $\Delta \Phi_n$ is proportional to the out-of-plane displacement of the surface between states $a$ and $b$, and can be obtained from each pair of complex-valued maps $(I^a_n, I^b_n)$ by computing\(^5\)

\[
\Delta \Phi_n = \Phi^b_n - \Phi^a_n = \phi^b_{o,n} - \phi^a_{o,n} = \text{arg}[I^b_n (I^a_n)^*]
\]  

(11)
where * stands for complex conjugation. The constant phase terms and the carriers are also removed in the process A sequence of 8 optical phase change maps was thus obtained, with \( n=15,\ldots,22 \). The 2DFT method— which would be equivalent to the 3DFT if the 3D filter were applied to all the temporal frequency planes, including \( n=0 \)— was applied to the same set of interferograms, and another sequence of 8 optical phase-change maps was obtained.

### 3.4 Maps of mechanical complex amplitude

From each \( \Delta \Phi_n \) obtained with the 2DFT and the 3DFT, a complex optical phase-change map \( \overline{\Delta \Phi_n}(x,t) \) that is proportional to the mechanical complex amplitude of the wavetrain was calculated. The procedure is summarized in Figure 5, (b) to (d). The modulus of the complex optical phase-change is \( \text{mod}[\Delta \Phi(x,t)]=4\pi/\lambda u_{3m,n}(x,t) \), where \( u_{3m,n}(x,t) \) is the amplitude of the out of plane displacement of the surface at map \( n \) due to the ultrasonic acoustic wave (i.e., the acoustic amplitude). Several optical phase-change maps \( \Delta \Phi_n \) and acoustic amplitude maps \( u_{3m,n} \) yielded by the 3DFT and the 2DFT methods are compared in the following section.

### 4. RESULTS

Figure 6 compares several experimental results obtained with the 3DFT to equivalent ones obtained with the 2DFT. Figures 6(i,a) and 6(i,b) show the optical phase-change maps \( \Delta \Phi_n \) at \( n=16 \) and \( n=19 \) (\( \Delta \Phi_{16} \) and \( \Delta \Phi_{19} \)), both obtained with the 2DFT method. Figures 6(ii,a) and 6(ii,b) show the corresponding optical phase-change maps \( \Delta \Phi_{16} \) and \( \Delta \Phi_{19} \) obtained with the 3DFT method. Though the wavetrain can be seen in all maps, those obtained with the 3DFT are less noisy. A noise rejection filter and contrast enhancement were applied to all the images to improve their visualization. The other four images in the same figure are 3D representations of the acoustic amplitude \( u_{3m,n}=[\lambda/(4\pi)]\text{mod}(\overline{\Delta \Phi_n}) \); these acoustic amplitude maps were obtained from the optical phase-change maps \( \Delta \Phi_n \) of columns (a) and (b) by following the procedure explained in section 3.4. Therefore, figures 6(i,c) and 6(i,d) correspond to the acoustic amplitudes for \( n=16 \) and \( n=19 \) (\( u_{3m,16} \) and \( u_{3m,19} \)) obtained with the 2DFT method. Figures 6(ii,c) and 6(ii,d) correspond to the acoustic amplitudes \( u_{3m,16} \) and \( u_{3m,19} \) obtained with the 3DFT method. In general, we observe that the improvement obtained with the 3DFT is apparent.

### 5. DISCUSSION

In our experiments, the strong spread of the interferential lobes in the temporal frequency axis causes that a small portion of their spectral content remains at plane \( n=0 \). This is barely observable in Fig. 4(a) because the modulus of the residual lobes at plane \( n=0 \) is negligible compared to that of the spectrum of the object beam. This portion of the spectrum of the side lobes is removed with the 3D filter, which introduces a phase error in the resultant optical phase maps. The fluctuation of the phase of the laser pulses arises as the most probable cause of this spread, so we believe that this phase error could be avoided by using a more stable laser source and/or a faster camera. A faster acquisition would also relax the requirements of isolation from environmental perturbations typically associated with the recording of a long sequence of consecutive images. Slight changes of intensity along the sequence of interferograms were discarded as a possible cause of the spectral broadening since, in one occasion, the intensities were equalized before applying the 3DFT.
The dynamics of the object and an imperfect phase shifting were also discarded, because the broadening was present even with the object at rest and the phase stepper turned off. In any case, since the spectrum of the object beam was thoroughly confined to plane $n'=0$, it was still feasible to remove it by means of an appropriate filter, even in presence of phase instability of the laser.

Finally, as long as the speckle is resolved by the camera and the interferential terms are completely separated from one another by the spatial carrier, the proposed method would allow the use of large apertures that increase the spatial resolution and the amount of light reaching the sensor. Also, in experiments where the recording of sequences of interferograms has interest _per se_, (e.g., in the study of time varying phenomena) its implementation would require only an additional effort of phase modulation.

### 6. CONCLUSIONS

In single interferograms that are processed with a 2D spatial Fourier transform to extract the optical phase, an overlap between the spectra of the interferential terms and the spectrum of the object beam may occur, which introduces noise in the resultant phase maps. In this work we present an acquisition and processing method aimed at removing this noise. The method requires the recording of a sequence of digital speckle interferograms with temporal and spatial carriers. Then, a 3D Fourier transform is applied to this set of spatiotemporal data. In the resultant 3D spectrum, the term related to the object beam spans a wide range of spatial frequencies but is confined to low temporal frequencies. The temporal carrier shifts the central temporal frequency of the interferential terms to high temporal frequencies, thus eliminating or minimizing their overlap with the low frequency terms. The role of the spatial carrier is to prevent a possible overlap of the conjugate interferential lobes in case they have a broadband temporal spectral content. A 3D filter, in the general case, is used to reject the low temporal frequencies and select one of the spatial side lobes. Finally, a sequence of complex-valued phase maps that contain the optical phases of the interferograms is obtained after the application of a 3D inverse Fourier transform. Experimental results of the measurement of the mechanical amplitude of guided acoustic waves in a metallic sample, obtained with a double-pulsed TV holography setup, are presented to illustrate the method. An apparent improvement is achieved with this new 3DFT method compared to the regular spatial-carrier 2DFT even in presence of phase instability of the laser.
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