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Measurement of Lamb waves dispersion curves under narrowband monomode excitation using TV holography

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ABSTRACT

Ultrasonic Lamb waves provide a useful means for the nondestructive determination of the material elastic constants of shell structures such as plates, pipes, cans and many others. A new optical technique is described for the measurement of the dispersion curves of Lamb wave modes. The experimental system employs the wedge method for the excitation of Lamb modes in aluminum plates of thickness in the range of a few millimetres. Long tone-bursts are used in order to ensure the generation of narrowband ultrasonic waves. Furthermore, an appropriate selection of the wedge angle allows one to generate only the desired individual Lamb mode. The detection of the surface out-of-plane displacements is performed by our self-developed pulsed TV holography system, which evaluates the optical phase by the Spatial Fourier Transform Method. Inasmuch as a whole-field measurement is realized, the wavelength of the excited mode can be precisely measured from the TV holography displacement maps. On the other hand, the wave frequency is measured by a pointwise method, namely a Michelson speckle interferometer. The phase velocity is directly obtained as the product of these two values. Measurements are done for several frequencies and several Lamb modes, thus yielding a collection of experimental points. By fitting these results to the theoretical Rayleigh-Lamb frequency spectrum, values of the shear wave velocity and the Poisson's ratio of the plate material are obtained. For a better accuracy in the measurements, the longitudinal phase velocity was directly determined by the pulse-echo method. The additional knowledge of the mass density allows one to calculate the Young's modulus.

Keywords: Pulsed TV holography, ESPI, Elastic Constant, Lamb Wave

1. INTRODUCTION

Ultrasound plays a very important role in the scope of non destructive testing. As a matter of fact, the use of ultrasonic waves for the elastic characterization of engineering structures is well established. The main part of these techniques rely on the measurement of longitudinal and shear wave velocities, \( c_L \) and \( c_T \) respectively, from which, provided the knowledge of the material mass density \( \rho \), one can obtain both Young's modulus \( E \) and Poisson's ratio \( \nu \) [1,2].

In plate structures, even though \( c_L \) and \( c_T \) can be directly measured by time-of-flight techniques, the usable frequency range for these measurements is limited by the fact that the wavelength must be shorter than the thickness of the plate. This limitation can be overcome by the use of Lamb waves, which present other advantages as well, such as their low attenuation along the plane of the plate (thus allowing a wider area of inspection) and their capability to measure both in plane and out-of-plane material properties. Moreover, their dispersive and multimode nature allows, by a proper selection of the mode and frequency, to improve the sensitivity of the measurements.

Several authors have investigated the use of Lamb waves for the elastic constants measurement of plate structures and it has been reported the employment of both narrowband and broadband excitation [3,4]. Broadband signals cover a large range of frequencies and commonly several Lamb modes; however, they require careful data processing in order to isolate the contribution to the signal of each mode and to fit them to the theoretical frequency spectrum. Narrowband signals, conversely, lack of this amount of information but the signal to noise ratio is much higher and a better accuracy is obtained in each experimental point.

Even using narrowband excitation several modes are normally excited and a beating is produced. With our experimental system we are able to detect this beating effect and in a former work we presented a method for determining the elastic properties of plates several millimetres thick. For this purpose we generated the A0 and S0 Lamb modes together in the plate and measured their beat wavelength [5].


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In this work we aim to excite narrowband single modes in the same plates of [5] and to determine their elastic constants from the experimental measurements of the wavelength of each individual Lamb mode. This parameter is obtained in the out-of-plane displacements maps rendered by our self-developed pulsed TV holography system.

2. THEORY

2.1. Lamb waves

Longitudinal and shear waves are the only kind of acoustic waves that can exist within an isotropic and homogeneous infinite medium. Their phase velocities, \( c_L \) and \( c_T \) respectively, are a consequence of the elastic constants of such medium. Conversely, due to the presence of boundaries or any other kind of nonhomogeneities, reflection and refraction phenomena can produce the onset of certain wave modes. That is the case of Lamb waves, which are generated on plates infinite in extent and having parallel stress-free boundaries. It is well known that for a certain frequency only a finite number of these modes can exist, namely those satisfying the condition given, in adimensional form, in the following equation [2]:

\[
F_{sym} = \frac{\tan \frac{\pi}{2} \sqrt{\gamma^2 - \xi^2}}{\tan \frac{\pi}{2} \sqrt{\gamma^2 - \xi^2}} + \left[ \frac{4 \xi^2 \sqrt{\gamma^2 - \xi^2}}{(2 \xi^2 - \gamma^2 \chi^2)} \right] = 0 \tag{1}
\]

where:

- Normalized wavenumber:
  \( \xi = \frac{2h}{\pi} k_1 \) \tag{2}

- Normalized frequency:
  \( \gamma = \frac{2h}{\pi} \frac{\omega}{c_L} \) \tag{3}

- Velocity ratio:
  \( \chi = \frac{c_L}{c_T} \) \tag{4}

and \( k_1 \) is the Lamb wavenumber, \( 2h \) the plate thickness and \( \omega = 2\pi f \) the circular frequency. In linear Elasticity, the Poisson's ratio \( \nu \) is a function of the velocity ratio:

\[
\nu = \frac{\chi^2 - 2}{2(\chi^2 - 1)} \tag{5}
\]

and the Young's modulus \( E \) can be calculated as:

\[
E = 2 \rho c_T^2 (1 + \nu) = \rho c_L^2 \left(\frac{(1 + 2\nu)(1 - 2\nu)}{1 - \nu}\right) \tag{6}
\]

The numerical resolution of Eq. 1 yields the relation between the frequency and the wavenumber of each mode as displayed in the frequency spectrum of Fig 4. The phase velocities can be calculated as the quotient of the two former magnitudes, and so the dispersion curves of Fig 4 are obtained.

The instantaneous out-of-plane displacements \( u_2 \) of the surface points along the propagation direction \( x_1 \) is given by

\[
u_2(x_1,t) = A \cos(k_1 x_1 - \omega t + \phi_0) \tag{7}
\]

2.2. TV holography

The measurement of the wavelength of each Lamb mode was performed by using our self-developed variant of a well-established optical technique called TV holography (TVH). We employed a double-pulsed TVH system that we have previously reported [6] and whose layout is depicted in Fig. 1. Our method is based on the recording of two speckle interferograms delayed an odd number of half periods -typically three- of the Lamb wave that propagates along the plate. These correlograms are processed following the procedure described in [7], thus obtaining the 2D map of the optical phase change, \( \Delta \Phi \), between the two pulses, which is proportional to the displacement field \( u_2 \) at the instant \( t_1 \) corresponding to the first laser pulse:

\[
\Delta \Phi(x_1,t_1) = \frac{8\pi}{\lambda} u_2(x_1,t_1) \tag{8}
\]
A second processing method, also described in [7], is applied to the optical phase change map in order to isolate the amplitude and phase of the acoustic wave. The acoustic amplitude map allows confirming the monomode nature of the excited wave, since a beating would be detected if two or more modes were present in the plate [5]. On the other hand, the wavelength of the Lamb wave can be measured more accurately in the acoustic phase map than in the optical phase change map, because the modulation of the fringe amplitude due to the burst envelope is not present in the first one.

3. EXPERIMENT

The excitation of the desired wave modes is performed by means of the prismatic coupling block method, also known as the wedge method, using long tone bursts of constant amplitude and sinusoidal profile. This technique offers a good repeatability and a fine control of the frequency and shape of the acoustic wave train.

We determined the elastic constants of the plate from the measured values of the wavelength and the frequency of individual Lamb wave modes. Therefore it is important to ensure that the waves in the plate are both narrowband and monomode.

The narrowband condition is rather easily achieved by using long tone bursts. The only limitation for the length of a burst is that the elapsed time between its beginning and the measurement instant must be short enough to avoid the presence in the displacement map of any reflection at the edges of the plate. In any case, we verified the narrow bandwidth of the waves using a speckle point Michelson interferometer, with which we recorded the instantaneous out-of-plane displacement of an area of the plate much smaller than the Lamb wavelength. Typical displacement temporal records are represented in Fig. 2, together with the modulus of their temporal Fourier transform. As we can see, the quasi-monochromatic nature of the waves is guaranteed even in the case of multimode waves.

On the contrary, the finite size of the ultrasonic source, i.e. the wedge, makes the monomode condition not so readily accomplished [8]. To excite a pure single mode, one should use monochromatic excitation and a wedge with an extent in the wave propagation direction large compared to the Lamb wavelength, and whose angle \( \theta \) satisfies the following condition:

\[
\cos \theta = \frac{c_p}{c_w} \tag{9}
\]

where \( c_p \) is the phase velocity of the Lamb mode and \( c_w \) is the longitudinal phase velocity of the material of the wedge. Nonetheless, when a relatively small wedge is used, any conceivable mode at the driving frequency is generated, the closer its phase velocity is to \( c_p \), the larger amplitude it has [8]. Therefore, provided \( c_p \) is close enough to the phase velocities of two or more modes, a beating effect is clearly detected that prevents the wavelength of an individual mode to be measured. The beating effect is shown in Fig 3(a,ii), where it is displayed the out-of-plane acoustic amplitude map when the modes A1 and S1 are together present in the plate. In order to excite single modes we carefully selected the driving frequency and the angle of the wedge.
Fig. 2. Out-of-plane instantaneous displacements in a point of the plate recorded by means of a Michelson interferometer. Propagation of mode A0 in a plate of 1.51mm, instantaneous displacement (a) and frequency spectrum (c). Propagation of modes A1 and S1 in a plate of 5.00mm, instantaneous displacement (b) and frequency spectrum (d).

Fig. 3 shows some of the experimental results obtained with our TVH system described in section 2.2. The optical phase change map, the acoustic amplitude map and the acoustic phase map are displayed in the first, second and third column respectively. We generated several pure modes in each plate and measured their wavelengths. These values are represented in Fig. 4 together with the theoretical frequency spectrum and dispersion curves for the calculated Poisson's ratio and bulk wave velocities in each plate.

The measurement of the wavenumbers of two or more modes in each plate suffices, in principle, to determine both $c_L$ and $\nu$ and hence any other elastic constant [3]. However, since we obtained a poor accuracy in the solutions, we opted to measure $c_L$ by the pulse-echo method. Therefore, once $c_L$ was known, we only had to calculate $\nu$ in order to characterize the plates. This resolution of the problem yielded a better accuracy than the former method described in [5]. The final results are displaced in Table 1 for the plates of thicknesses 2.98, 4.03 and 5.00mm.

4. CONCLUSIONS

The wavelengths of individual Lamb modes have been measured by pulsed TV holography. These experimental values were processed in order to obtain the elastic constants of several aluminum plates. Since the accuracy of the results was not good, another experimental parameter was taken into account, namely the longitudinal wave velocity $c_L$. With this additional information the elastic parameters of the plates were obtained with an estimated accuracy better than ±4%.

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Fig. 3. Visualization of waves with pulsed TVH. (i) Optical phase change maps. (ii) Acoustic amplitude maps. (iii) Acoustic phase maps. (a) Beating of modes A1 and S1 at 1MHz in a plate of 2.98mm. (b) Mode S0 at 0.66MHz in a plate of 2.98mm. (c) Mode A1 at 1.05MHz in a plate of 4.03mm. (d) Mode A1 at 0.8MHz in a plate of 5.00mm.

Table 1. Measured elastic constants of the plates

<table>
<thead>
<tr>
<th>2h (mm)</th>
<th>c_L (m/s)</th>
<th>ν</th>
<th>c_T (m/s)</th>
<th>ρ (kg/m³)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.98±0.01</td>
<td>6330±30</td>
<td>0.3390±0.0085</td>
<td>3124±62</td>
<td>2790±15</td>
<td>72.9±2.6</td>
</tr>
<tr>
<td>4.03±0.01</td>
<td>6330±30</td>
<td>0.3410±0.0078</td>
<td>3109±60</td>
<td>2790±15</td>
<td>72.3±2.4</td>
</tr>
<tr>
<td>5.00±0.01</td>
<td>6315±30</td>
<td>0.3398±0.0062</td>
<td>3111±48</td>
<td>2790±15</td>
<td>72.3±2.0</td>
</tr>
</tbody>
</table>
Fig. 4. Experimental points fitting the theoretical frequency spectrum (a)(b)(c) and the dispersion curves (d)(e)(f) for the calculated elastic constants of each plate. (a)(d) Plate of 2.98mm. (b)(e) Plate of 4.03mm. (c)(f) Plate of 5.00mm.

REFERENCES