Measurement of the mechanical amplitude and phase of transient surface acoustic waves using double-pulsed TV Holography and the spatial Fourier transform method

Cristina Trillo, Ángel F. Doval*, Daniel Cernadas, Oscar López, Carlos López, Benito V. Dorrio, Jose L. Fernández and Mariano Pérez-Amor


ABSTRACT
We present a technique to calculate the mechanical amplitude and phase of an ultrasonic plane wavefield of nanometric amplitude that propagates on a surface. Our aim is to detect perturbations of the initially smooth wavefronts that indicate the presence of flaws in the material. We use bursts of Rayleigh waves and a double-pulsed TV Holography system that records two correlograms separated down to 1.5 microseconds. The phases of the correlograms are calculated separately using the spatial Fourier transform method (SFTM), and subtracted. In the resultant phase map, the field of instantaneous displacements of the surface (that comprises several periods of the surface acoustic wave) acts as a modulated spatial carrier, now related to the mechanical phase and amplitude, that are extracted applying the SFTM again.

Keywords: Pulsed TV Holography, transient surface acoustic waves, mechanical amplitude and phase, spatial Fourier transform method

1. INTRODUCTION
Electronic speckle pattern interferometry (ESPI) also known as TV Holography (TVH), is a well-established whole-field optical technique. It has been successfully used in a wide range of applications, such as surface contouring, vibration analysis and non-destructive testing. In the field of vibration analysis, the measurement of the amplitude and phase of vibrations was carried out with a wide variety of illumination schemes and phase evaluation methods.1-4 The advent of pulsed lasers made it possible the study of transient events. Some authors have calculated the instantaneous surface displacement associated to transient events using pulsed illumination and phase evaluation methods like spatial phase stepping,5 spatial synchronous detection6 and the Fourier transform method7 (FTM).8 In this work we calculate the mechanical amplitude and phase of vibration of bursts of surface acoustic waves (SAW) by means of a previously reported double-pulsed TVH system9 and by applying twice the FTM: first to find the optical phase-change that measures the displacement of the surface between pulses and second to find the mechanical amplitude and phase of the SAW. An off-axis reference beam yields the optical spatial carrier and the SAW itself acts as a mechanical spatial carrier for the FTM.10 We just need two single-exposure interferograms recorded with a very short time delay between them to characterize completely non-repetitive transient phenomena with spatial periodicity. Our interest in SAW is justified because they have proven to be a valuable tool in non-destructive testing, in combination with both stroboscopic11 and pulsed12 TVH.

2. THEORY

Geometry Fig. 1 shows the layout that we have used in our experiments.9 The main elements are a pulsed light source, an interferometer, a digital camera, a SAW generator and custom electronics for the synchronization of the former devices. The interferometer is currently arranged to detect out-of-plane displacements, thus the object phase difference induced by the measurand is

\[ \phi_o = -\frac{4\pi}{\lambda} z \]  

*E-mail: adoval@uvigo.es

where $\lambda$ is the wavelength of the laser light and $z$ is the out-of-plane component of the displacement of the surface. We have chosen the direction $z > 0$ from the test piece towards the camera.

**Temporal treatment** The camera records two single-exposure interferograms of two states of the deformed surface of the object at the instants $t_n$ ($n = 1, 2$). To apply the spatial Fourier transform method, the reference beam has been shifted off the optical axis. It introduces a spatial carrier in the interferograms recorded by the camera, that we will refer to as *primary correlograms*, whose intensities can be expressed as follows

$$I_n = gI_{0,n} \left[ 1 + V_n \cos \left( \psi_{p,n} + \phi_{r,n} - \phi_{r,n} + 2\pi f_c x \right) \right]$$

where $I_n = I_n(x)$ is the primary correlogram corresponding to the $n$-th pulse, $g = g(\lambda)$ is the spectral sensitivity of the camera, $I_{0,n} = I_{0,n}(x)$ is the local central value of the intensity, $V_n = V_n(x)$ is the local visibility, $\psi_{p,n} = \psi_{p,n}(x)$ is the random phase difference of the interfering speckle patterns, $\phi_{r,n} = \phi_{r,n}(x, t_n)$ is the reference phase difference, $f_c = (f_x, f_y)$ is the frequency of the spatial carrier and $x = (x, y)$ is the position on the image plane.

The surface displacement $z(x, t_n)$ of an acoustic wave that propagates on the surface of a slab or plate in the instant $t_n$ can be written

$$z = z_m \cos (\varphi_M + k_M \cdot x + \varphi_n)$$

where $z_m = z_m(x, t_n)$ is the amplitude, $\varphi_M = \varphi_M(x)$ is the initial phase delay, $k_M = 2\pi/\lambda_M$ is the wavevector (scaled to the image plane) in the considered region, $\lambda_M$ is the wavelength and $\omega_M$ is the angular frequency of the SAW, and $\varphi_n = \varphi_n(t_n) = -\omega_M t_n$, being $t_n$ the delay of the illumination pulses within each vibration period.

From Equations 1, 2 and 3

$$I_n = gI_{0,n} \left[ 1 + V_n \cos [\psi_{p,n} - \phi_{om,n} \cos (\varphi_M + k_M \cdot x + \varphi_n) - \phi_{r,n} + 2\pi f_c \cdot x] \right]$$

with

$$\phi_{om,n} = \frac{4\pi}{\lambda} z_m$$
We rewrite Eq. 4 as follows

\[ I_n = g I_{0,n} [1 + V_n \cos(\Phi_n + \Phi_c)] \]  

(6)

where \( \Phi_c = 2\pi f_c \cdot x \) is the phase of the spatial carrier and \( \Phi_n \) follows the expression

\[ \Phi_n = \psi_{p,n} - \phi_{om,n} \cos(\varphi_M + k_M \cdot x + \varphi_n) - \phi_{r,n} \]  

(7)

**Surface displacement calculation by applying the spatial Fourier transform method**  
In this first stage of our technique we follow the method proposed by Saidner et al.\(^{13}\) for TVH that calculates the phase change of the image field between exposures. After the computation of the Fourier transform, the filtering of one of the shifted terms of the spectrum and the calculation of the inverse Fourier transform, the expression of each primary correlogram becomes

\[ \hat{I}_n = \frac{1}{2} g I_{0,n} \exp[j(\Phi_n + \Phi_c)] \]  

(8)

We obtain directly the phase difference from a formula published originally by Stetson.\(^{14}\) It follows the expression

\[ \Delta \Phi = \arctan \frac{\text{Re}(\hat{I}_1) \text{Im}(\hat{I}_2) - \text{Im}(\hat{I}_1) \text{Re}(\hat{I}_2)}{\text{Im}(\hat{I}_1) \text{Im}(\hat{I}_2) + \text{Re}(\hat{I}_1) \text{Re}(\hat{I}_2)} \]  

(9)

which yields

\[ \Delta \Phi = [\psi_{p,2} - \phi_{om,2} \cos(\varphi_M + k_M \cdot x + \varphi_2) - \phi_{r,2}] - [\psi_{p,1} - \phi_{om,1} \cos(\varphi_M + k_M \cdot x + \varphi_1) - \phi_{r,1}] \]  

(10)

As it is usual in TVH techniques, we assume that \( \psi_2 = \psi_{p,2} = \psi_{p,1} \) and, since there is not modulation of the phase of the reference beam, we can write \( \phi_r = \phi_{r,2} = \phi_{r,1} \). We use a wave burst that is long enough to cover entirely the region under inspection. In this case the amplitude of vibration of the SAW can be considered to depend solely on the position on the image plane, that is, \( \phi_{om,n} = \phi_{om}(x) \). After these simplifications, Eq. 10 becomes

\[ \Delta \Phi = -\phi_{om} [\cos(\varphi_M + k_M \cdot x + \varphi_2) - \cos(\varphi_M + k_M \cdot x + \varphi_1)] \]  

(11)

The maximum surface displacement between pulses is attained when \( \Delta \varphi \) is given the general value\(^{9}\)

\[ \Delta \varphi = \varphi_2 - \varphi_1 = (2q + 1) \pi, \quad q = 0, \pm 1, \pm 2... \]  

(12)

Due to the characteristics of our setup, \( q = 1 \) in our experiments.\(^9\) Substituting this expression in Eq. 11, we obtain

\[ \Delta \Phi = 2\phi_{om} \cos(\varphi_M + k_M \cdot x + \varphi_1) = \frac{8\pi}{\lambda} z_m \cos \left( \frac{\varphi_M}{\lambda M} + \frac{1}{\lambda M} x + \varphi_1 \right) \]  

(13)

**Calculation of the mechanical amplitude and phase of the surface acoustic wave**  
The phase map given by Eq. 13 contains the actual mechanical phase of the surface acoustic wave into the argument of a cosine function that can be interpreted as a modulated spatial carrier (see Fig. 2 (c)). We compute the Fourier transform of Eq. 13 and apply a suitable filter. The inverse Fourier transform of the result yields the complex amplitude of the surface wave as follows

\[ A = \frac{4\pi}{\lambda} z_m \exp \left[ j \left( \varphi_M + 2\pi \frac{1}{\lambda M} x + \varphi_1 \right) \right] \]  

(14)

from which we can obtain pointwisely its modulus and argument.

\[ \text{mod} (A) = \sqrt{\text{Re}^2 (A) + \text{Im}^2 (A)} = \frac{4\pi}{\lambda} z_m \]  

(15)

\[ \arg (A) = \arctan \frac{\text{Im} (A)}{\text{Re} (A)} = \varphi_M + 2\pi \frac{1}{\lambda M} x + \varphi_1 \]  

(16)
3. EXPERIMENTAL

Fig. 1 shows the system we have used in our experiments. The light source is a twin cavity Nd:YAG pulsed laser, which produces 25 double pulses per second. Both cavities have a common injection seeder to increase their self and mutual coherence and their outputs are combined and frequency doubled. The laser yields a resultant green laser radiation of $\lambda = 532$ nm that doubles the sensitivity of the interferometer and is detectable with standard CCD cameras. The laser output radiation is divided into an object beam, which illuminates the object surface after being expanded by a diverging lens, and a reference beam, which is guided by optical fiber in order to give some degree of mobility to the head. We shift the reference beam off the optical axis to obtain a suitable spatial carrier in the primary correlograms. In the reference arm there are also a polarizer and a polarization controller that, conveniently adjusted, help to maximize the visibility of the primary correlograms. The light scattered by the object and that of the reference beam are combined in a beam splitter and are imaged onto the photosensitive surface of the CCD camera. It can record two images with full spatial resolution (1280 x 1024 pixels) separated down to 1 s, it is thermoelectrically cooled and digitizes each image with a resolution of 12 bits. The SAW are Rayleigh waves produced by a programmable burst wave generator and are coupled to a slab by means of the well-known prismatic coupling block method. The coupled Rayleigh waves propagate at the measured velocity of 2976 ± 60 m/s and the nominal frequency of the pulses in each burst is 1 MHz. A previous independent measurement using a point speckle Michelson interferometer let us know that the SAW produced with our wedge system on an aluminium slab have out-of-plane amplitudes near 10 nm. In our experiments, the laser pulses were separated by the minimum number of half-periods allowed by the camera (three half-periods, what gives $\Delta t = t_2 - t_1 = 1.5 \mu s$) in order to preserve the actual shape of the wave burst and to reduce the effects of environmental noise as much as possible.

The laser, the SAW generator and the camera are synchronized by means of a computer, two commercially available delay generators and a custom electronic stage. The remote operation of all the devices, as well as the digital processing of the correlograms, was accomplished with a personal computer and a specific software application developed by our group. For the calculation of the Fourier transforms we have used an standard FFT algorithm.

4. RESULTS AND DISCUSSION

Fig. 2 illustrates the technique developed in Section 2 using six of the most representative images obtained throughout the process. Fig. 2(a) shows one primary correlogram with spatial carrier and Fig. 2(b) the corresponding Fourier transform spectrum. The chosen side lobe is bounded by a white rectangle that marks the size and position of the filter. Fig. 2(c) shows the phase change of the image field between correlograms, which corresponds to Eq. 13. To give prominence to the presence of the SAW we calculate the average phase $m$ and standard deviation $\sigma$ of the pixels of the image whose intensity level is higher than a given threshold value, and then we highlight the ones whose phase belongs to the interval $m \pm \sigma$. This operation is performed only for display purposes and does not change the data used in subsequent mathematical operations. Fig. 2(d) shows the Fourier transform of the phase map in 2(c) and a white rectangle denotes the filter, similarly to Fig. 2(b). The result agrees with the Fourier transform of a sinusoidal function, as one might expect. The filters applied to the images in 2(b) and 2(d) are both two-dimensional Hanning window functions. Finally, Figs. 2(e) and 2(f) show the amplitude and phase of the SAW. The amplitude distribution agrees with the expected behaviour of Rayleigh waves, i.e. the amplitude is maximum in the center of the burst (maximum brightness in the image) and it does not decay substantially in short propagation distances. Fig. 3 shows the instant displacement (a), the amplitude (b) and the phase (c) of a long burst of Rayleigh waves that propagates on a slab with a subsurface bore. The effect of the flaw is very apparent in the three images. Fig. 3(b) shows that the amplitude of the Rayleigh wave seems to decrease as the propagation distance increases. The larger field of view with regard to Fig. 2 and the diffraction induced by the flaw may explain this effect.

5. CONCLUSIONS

We present a double-pulse TVH technique that permits to calculate the mechanical amplitude and phase of non-repetitive transient phenomena with spatial periodicity by using two single-exposure correlograms recorded with a very short time delay between them. We combine this technique with bursts of SAW because of their applications in the field of non destructive-testing. We evaluate the mechanical amplitude and phase of SAW that propagate on both defect-free and faulty slabs. The presence of the flaw is clearly detected in the output images herein presented, and the perturbation is specially apparent in the image that displays the mechanical amplitude of the SAW. This feature might be exploited to develop future flaw-detection techniques.
Figure 2. Primary correlogram (a) and its Fourier transform spectrum (b). Phase difference between exposures (c) and its Fourier transform (d). The white rectangles in (b) and (d) mark the size and position of the respective filters. Amplitude (e) and phase (f) of the SAW. The SAW are Rayleigh waves that propagate on a slab without flaws. The size of the field of view is 58 mm × 58 mm.

Figure 3. Optical phase difference corresponding to surface displacement between exposures (a), amplitude (b) and phase (c) of a long burst of Rayleigh waves. The burst propagates from right to left over a subsurface defect (bore). The circular black mark in (a) indicates the vertical projection of the bore. The dark shadow on the right side of the image (a) is the wedge. The size of the field of view is 76 mm × 76 mm.
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