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http://dx.doi.org/10.1117/1.OE.55.12.121709
Propagation of the measurement uncertainty in Fourier transform digital holographic interferometry

Ángel F. Doval
Cristina Trillo
J. Carlos López-Vázquez
José L. Fernández

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Ángel F. Doval,* Cristina Trillo, J. Carlos López-Vázquez, and José L. Fernández
Universidad de Vigo, Departamento de Física Aplicada, Escuela de Ingeniería Industrial, Campus Universitario, Vigo 36310, Spain

Abstract. The derivation of expressions to evaluate the local standard uncertainty of the complex amplitude of the numerically reconstructed field as well as of the phase-change measurements resulting from Fourier- and quasi-Fourier transform digital holographic interferometry is presented. Applying the law of propagation of uncertainty, as defined in the “Guide to the expression of uncertainty in measurement,” to the digital reconstruction of holograms by Fourier transformation and to the subsequent calculation of the phase change between two such reconstructions results in a set of expressions, which allow the evaluation of the uncertainties of the complex amplitude and of the phase change at every pixel of the reconstruction in terms of the measured values and their standard uncertainty in the pixels of the original digital holograms. These expressions are increasingly simplified by first assuming a linear dependence between the squared uncertainty and the local value of the original holograms, and then considering that the object beam is a speckle pattern. We assess the behavior of the method by comparing the predicted standard uncertainty with the sample variance obtained from experiments conducted under repeatability conditions, and find a good agreement between both quantities. © 2016 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: 10.1117/1.OE.55.12.121709]

Keywords: digital holography; interferometry; uncertainty; phase measurement.

1 Introduction

High-performance measurement techniques—and, among them, digital holographic interferometry—require a method to estimate the uncertainty for each measured value they yield. The “Guide to the expression of uncertainty in measurement” (GUM) specifies two ways to evaluate the uncertainty of measurement: type A evaluation, where the measurement uncertainty is estimated by a statistical analysis of multiple values of the measured quantity obtained under repeatability or other well-defined measurement conditions and, on the other hand, type B evaluation, where the uncertainty is determined by means other than direct statistical analysis, such as using a measurement model and prior knowledge, specifications, calibration data, and so on.

Type A evaluation of uncertainty can definitely be applied to digital holographic interferometry, but in many practical situations—such as in industrial or noncontrolled environments, when measuring dynamic or transient events, and so on—it is not possible to get repeated measurements in the same conditions, and type B evaluation of uncertainty becomes necessary. Digital Fourier transform holograms, including lensless and quasi-Fourier transform holograms, can be reconstructed solely by calculating their Fourier transforms. This relative simplicity makes them particularly suitable for a first approach to type B uncertainty evaluation in digital holographic interferometry.

The analysis of the overall accuracy of digital holographic reconstruction and phase-change measurements has been approached with both type A and type B global evaluation methods. The goal of this work is to derive expressions to get a type B evaluation of the local standard uncertainty of both the complex amplitude of the reconstructed field and the phase-change maps resulting from the application of single-exposure digital holographic interferometry techniques to Fourier transform holograms, as well as to verify that, under repeatability conditions, the estimations of the uncertainty calculated with the resulting expressions match those resulting of type A evaluation.

The present article is an extended version of the preliminary results presented at the “Speckle VI” conference. The theoretical aspects of the propagation of the uncertainty from the holograms to the complex amplitude of their reconstructions and to the phase change are presented in Sec. 2. The resulting expressions are increasingly simplified by first assuming a linear model for the camera noise and then taking into account the particularities of the presence of speckle in the object beam; this extended version includes approximated expressions for the standard uncertainties of the real and imaginary parts of the complex amplitude of the reconstructed holograms, in addition to those corresponding to the phase-change between holograms already reported in the conference proceedings. A new and larger set of experimental measurements, which is described in Sec. 3, has been conducted to test the validity of the resulting expressions of the uncertainty. In addition to measurements with different hologram exposure times, this new set incorporates repeated measurements with uncorrelated speckle patterns for each exposure time. These measurements have been processed to show the degree of agreement between the observed variances and the corresponding estimations of the squared standard uncertainty by following a procedure (Sec. 3.3), which is somewhat different from the approach taken in the aforementioned conference communication. On the one hand, the
values of the parameters of the hologram uncertainty model are now derived from the characterization of the camera with incoherent illumination according to the EMVA 1288 standard and, on the other hand, the analysis of the experimental data has been reformulated to assess the agreement between the estimations of the squared standard uncertainty and the sample variances of the real and imaginary parts of the holographic reconstructions in addition to those of the phase change. The results are eventually discussed in Sec. 4.

2 Theory

2.1 Propagation of Uncertainty in Fourier Transform Digital Hologram Reconstruction

A digital Fourier transform hologram, recorded using a camera with \( N \times M \) pixels of size \( \Delta x \times \Delta y \), is a matrix of positive real values:

\[
h = h(q, p) = h(q\Delta x, p\Delta y); \quad 0 < q < N, 0 \leq p < M.
\]

(1)

It is numerically reconstructed with a two-dimensional discrete Fourier transform, which can be defined as

\[
\mathcal{F}(h) = H = H(n, m) = H(n\Delta f_x, m\Delta f_y) = \Delta x\Delta y \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} h(q\Delta x, p\Delta y) \exp\left(\frac{i2\pi qn}{N}\right)
\]

\[
\times \exp\left(\frac{i2\pi pm}{M}\right) = \Delta x\Delta y \text{FFT}(h) = \Delta x\Delta y \hat{H}.
\]

(2)

where \( \Delta f_x = 1/(N\Delta x) \), \( \Delta f_y = 1/(M\Delta y) \), and the double sum is usually calculated by using the fast Fourier transform (FFT) of the hologram, which is defined as

\[
\hat{H} = \hat{H}(n, m) = \hat{H}(n\Delta f_x, m\Delta f_y) = \text{FFT}(h) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} h(q, p) \exp\left[i2\pi \frac{qn}{N} + \frac{pm}{M}\right],
\]

(3)

with real and imaginary parts, respectively,

\[
\text{Re} \hat{H} = \text{Re} \hat{H}(n, m) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} h(q, p) \cos\left[2\pi \frac{qn}{N} + \frac{pm}{M}\right],
\]

(4)

\[
\text{Im} \hat{H} = \text{Im} \hat{H}(n, m) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} h(q, p) \sin\left[2\pi \frac{qn}{N} + \frac{pm}{M}\right].
\]

(5)

The local standard uncertainty of the real and imaginary parts of the complex amplitude of the reconstructed holographic field can be calculated from the local standard uncertainty of the digital hologram, \( u[h(q, p)] \), by using the “law of propagation of uncertainty”\(^{11,10}\) as follows:\(^1\)

\[
u^2(\text{Re} \hat{H}) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} \left[ \frac{\partial \text{Re} \hat{H}(n, m)}{\partial h(q, p)} \right]^2 u^2(h(q, p))
\]

\[
= \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \left\{ 1 + \cos\left[4\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right]\right\}
\]

\[
= \frac{1}{2} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) - \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \cos\left[4\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right],
\]

(6)

\[
u^2(\text{Im} \hat{H}) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} \left[ \frac{\partial \text{Im} \hat{H}(n, m)}{\partial h(q, p)} \right]^2 u^2(h(q, p))
\]

\[
= \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \left\{ 1 - \cos\left[4\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right]\right\}
\]

\[
= \frac{1}{2} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) - \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \sin\left[4\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right],
\]

(7)

\[
u(\text{Re} \hat{H}, \text{Im} \hat{H}) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} \frac{\partial \text{Re} \hat{H}(n, m)}{\partial h(q, p)} \frac{\partial \text{Im} \hat{H}(n, m)}{\partial h(q, p)} u^2(h(q, p))
\]

\[
= \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \cos\left[2\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right]
\]

\[
\times \sin\left[2\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right]
\]

\[
= \frac{1}{2} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2(h(q, p)) \sin\left[4\pi \left(\frac{qn}{N} + \frac{pm}{M}\right)\right],
\]

(8)

and assuming that the uncertainties of \( \Delta x \) and \( \Delta y \) can be neglected

\[
u^2(\text{Re} H) = (\Delta x)^2(\Delta y)^2 u^2(\text{Re} \hat{H}),
\]

(9)

\[
u^2(\text{Im} H) = (\Delta x)^2(\Delta y)^2 u^2(\text{Im} \hat{H}),
\]

(10)

\[
u(\text{Re} H, \text{Im} H) = (\Delta x)^2(\Delta y)^2 u(\text{Re} \hat{H}, \text{Im} \hat{H}).
\]

(11)
Taking into account that the FFT of $u^2[h(q, p)]$ is

$$
\hat{U}_s(n, m) = \text{FFT}\{u^2[h(q, p)]\} = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2[h(q, p)] \exp\left[2\pi \frac{qn + pm}{MN}\right]
$$

$$
= \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2[h(q, p)] \cos\left[2\pi \frac{qn + pm}{MN}\right] + i \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} u^2[h(q, p)] \sin\left[2\pi \frac{qn + pm}{MN}\right],
$$

(12)

the uncertainties in Eqs. (6)–(8) can be rewritten as

$$
u^2[\text{Re } \hat{H}(n, m)] = \frac{1}{2} [\hat{U}_s(0,0) + \text{Re } \hat{U}_s(2n, 2m)],
$$

(13)

$$
u^2[\text{Im } \hat{H}(n, m)] = \frac{1}{2} [\hat{U}_s(0,0) - \text{Re } \hat{U}_s(2n, 2m)],
$$

(14)

$$
u[\text{Re } \hat{H}(n, m), \text{Im } \hat{H}(n, m)] = \frac{1}{2} \text{Im } \hat{U}_s(2n, 2m).
$$

(15)

### 2.2 Propagation of Uncertainty in the Calculation of the Phase Change

Let us now consider the digital reconstructions of two Fourier transform holograms

$$
H_i = H_i(n, m) = H_i(n\Delta f_x, m\Delta f_y)
= \Delta x\Delta y \bar{H}_i(n\Delta f_x, m\Delta f_y)
= \Delta x\Delta y \hat{H}_i(n, m) = \Delta x\Delta y \hat{H}_i ; i \in \{1, 2\}.
$$

(16)

#### 2.2.1 Calculation of the phases and their difference

The most straightforward approach to the calculation of the phase change between the reconstructed holograms consists in obtaining their respective random-distributed phases:

$$
\psi_i = \psi_i(n, m) = \arctan \frac{\text{Im } \hat{H}_i(n, m)}{\text{Re } \hat{H}_i(n, m)} \in (-\pi, \pi],
$$

(17)

subtracting $\psi_1$ from $\psi_2$ and eventually reducing the phase change to the principal interval $(-\pi, \pi]$ to remove the phase-wrap arising from the random components of the phases results in

$$
\phi_{12} = \phi_{12}(n, m) = \begin{cases} 
\psi_2 - \psi_1 + 2\pi & \text{if } -2\pi < \psi_2 - \psi_1 \leq -\pi \\
\psi_2 - \psi_1 & \text{if } -\pi < \psi_2 - \psi_1 \leq \pi \\
\psi_2 - \psi_1 - 2\pi & \text{if } \pi < \psi_2 - \psi_1 \leq 2\pi 
\end{cases}
$$

(18)

Applying the law of propagation of uncertainty to Eq. (18) and assuming that the two measurements of the phase are statistically uncorrelated—I.e., $u(\psi_1, \psi_2) = 0$—yields

$$
u^2(\phi_{12}) = \nu^2(\psi_1) + \nu^2(\psi_2),
$$

(19)

where the standard uncertainties of the phase measurements are estimated by further applying the law of propagation to Eq. (17). Let us take, for simplicity, the generic expression:

$$
\psi_i = \arctan \frac{b_i}{a_i},
$$

(20)

with $z_i = a_i + ib_i = \text{Re } \hat{H}_i + i\text{Im } \hat{H}_i = \hat{H}_i$. The square of the combined standard uncertainty of $\psi_i$ is

$$
u^2(\psi_i) = \left(\frac{\partial \psi_i}{\partial a_i}\right)^2 \nu^2(a_i) + \left(\frac{\partial \psi_i}{\partial b_i}\right)^2 \nu^2(b_i) + 2 \frac{\partial \psi_i}{\partial a_i} \frac{\partial \psi_i}{\partial b_i} \nu(u(a_i, b_i))
= \frac{b_i^2}{|z_i|^2} \nu^2(a_i) + \frac{a_i^2}{|z_i|^2} \nu^2(b_i) - \frac{2a_i b_i}{|z_i|^2} \nu(u(a_i, b_i)).
$$

(21)

Incorporating Eq. (21) into Eq. (19) eventually results in

$$
u^2(\phi_{12}) = \frac{b_1^2}{|z_1|^2} \nu^2(a_1) + \frac{a_1^2}{|z_1|^2} \nu^2(b_1) + \frac{b_2^2}{|z_2|^2} \nu^2(a_2) + \frac{a_2^2}{|z_2|^2} \nu^2(b_2) - 2 \frac{a_1 b_1}{|z_1|^2} \nu(u(a_1, b_1)) - 2 \frac{a_2 b_2}{|z_2|^2} \nu(u(a_2, b_2)).
$$

(22)

#### 2.2.2 Direct calculation of the phase change with Stetson and Brohinsky’s algorithm

The Stetson–Brohinsky differential algorithm is a widely used alternative, which directly yields the value of phase change between the two reconstructed holograms constrained to the principal interval $(-\pi, \pi]$. Since it can be reasonably assumed that $\Delta x$ and $\Delta y$ are exactly the same at each pixel for both holograms,

$$
\phi_{12} = \psi_2 - \psi_1 = \arg H_{12} = \arg(H_1^*H_2)
= \arg(\Delta x\Delta y \bar{H}_1^* \Delta x\Delta y \bar{H}_2) = \arg(\hat{H}_1^* \hat{H}_2) = \arg(\hat{H}_{12}),
$$

(23)

$$
\phi_{12} = \arctan \frac{\text{Im } \hat{H}_{12}}{\text{Re } \hat{H}_{12}} = \arctan \frac{\text{Re } \hat{H}_1 \text{Im } \hat{H}_2 - \text{Re } \hat{H}_2 \text{Im } \hat{H}_1}{\text{Re } \hat{H}_1 \text{Re } \hat{H}_2 + \text{Im } \hat{H}_1 \text{Im } \hat{H}_2}.
$$

(24)

Let us simplify this expression by using the notation introduced in Eq. (20) to apply the law of propagation of uncertainty:

$$
\phi_{12} = \phi_{12}(z_1, z_2) = \phi_{12}(a_1 + ib_1, a_2 + ib_2)
= \phi_{12}(a_1, b_1, a_2, b_2) = \arctan \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2}.
$$

(25)

The square of the standard uncertainty of the phase change at a given point is, accordingly,
\[ u^2(\phi_{12}) = \left( \frac{\partial \phi_{12}}{\partial a_1} \right)^2 u^2(a_1) + \left( \frac{\partial \phi_{12}}{\partial b_1} \right)^2 u^2(b_1) \]
\[ + \left( \frac{\partial \phi_{12}}{\partial a_2} \right)^2 u^2(a_2) + \left( \frac{\partial \phi_{12}}{\partial b_2} \right)^2 u^2(b_2) \]
\[ + 2 \frac{\partial \phi_{12}}{\partial a_1} \frac{\partial \phi_{12}}{\partial b_1} u(a_1, b_1) + 2 \frac{\partial \phi_{12}}{\partial a_1} \frac{\partial \phi_{12}}{\partial a_2} u(a_1, a_2) \]
\[ + 2 \frac{\partial \phi_{12}}{\partial a_2} \frac{\partial \phi_{12}}{\partial b_2} u(a_2, b_2) + 2 \frac{\partial \phi_{12}}{\partial a_2} \frac{\partial \phi_{12}}{\partial a_2} u(a_2, a_2), \]
\[ \text{(26)} \]

which, assuming that \( u(a_i, b_j) = u(a_i, a_j) = u(b_i, b_j) = 0 \ \forall i \neq j \), i.e., \( z_1 = H_1 \) and \( z_2 = H_2 \) are statistically uncorrelated, can be written as

\[ u^2(\phi_{12}) = \frac{b_1^2}{|z_1|^4} u^2(a_1) + \frac{a_1^2}{|z_1|^4} u^2(b_1) + \frac{b_2^2}{|z_2|^4} u^2(b_2) \]
\[ \quad + \frac{a_1^2}{|z_2|^4} u^2(b_1) - 2 \frac{a_1 b_1}{|z_1|^4} u(a_1, b_1) - 2 \frac{a_2 b_2}{|z_2|^4} u(a_2, b_2). \]
\[ \text{(27)} \]

### 2.2.3 Application to the digital reconstructions of Fourier transform holograms

Equations (22) and (27) are the same and, therefore, the uncertainty of the measured phase change is the same regardless of which of the two methods is used to calculate it. The expression is particularized for the digital reconstructions of two holograms by substituting \( \tilde{H}_i = \text{Re} \tilde{H}_i + i \text{Im} \tilde{H}_i \) for \( z_j = a_j + ib_j \) and Eqs. (13)–(15) for the uncertainties into Eq. (27), resulting in

\[ u^2(\phi_{12}) = \frac{1}{2} \left[ \frac{1}{|H_1|^4} \tilde{U}_{1s}(0,0) + \text{Re} \tilde{U}_{1s}(2n, 2m) \right] \]
\[ + \frac{1}{2} \left[ \frac{1}{|H_2|^4} \tilde{U}_{2s}(0,0) + \text{Re} \tilde{U}_{2s}(2n, 2m) \right] \]
\[ + \frac{1}{2} \left[ \frac{\text{Im} \tilde{H}_1}{|H_1|^4} \tilde{U}_{1s}(2n, 2m) \right] \]
\[ - \frac{1}{2} \left[ \frac{\text{Re} \tilde{H}_1 \text{Im} \tilde{H}_1}{|H_1|^4} \text{Im} \tilde{U}_{1s}(2n, 2m) \right] \]
\[ - \frac{1}{2} \left[ \frac{\text{Re} \tilde{H}_1 \text{Im} \tilde{H}_2}{|H_2|^4} \text{Im} \tilde{U}_{2s}(2n, 2m) \right], \]
\[ \text{(28)} \]

where \( \phi_{12} = \phi_{12}(n, m) \) and \( \tilde{H}_i = \tilde{H}_i(n, m) \) are local, pixel-dependent, values.

### 2.3 Simplifications for Holograms with Linear Squared Standard Uncertainty

Since digital holograms are normally rendered by a digital camera, it can be reasonably assumed that the square of the standard uncertainty is linearly dependent with their local values \( h(q, p) \):

\[ u^2[h(q, p)] = kh(q, p) + u_0^2. \]
\[ \text{(30)} \]

This is consistent (see Sec. 3.2) with the noise model adopted in the EMVA 1288 camera characterization standard. Here, \( u_0 \) is a component of the standard uncertainty, which takes the same value for all of the pixels in the hologram. It typically models the uncertainty arising from quantization and temporal dark noises. On the other hand, \( kh(q, p) \) is proportional to the local value of the hologram, with the same value of the proportionality constant \( k \)—the overall system gain of the digital camera, typically expressed in counts per electron (DN/e\(^{-}\))—for all of the pixels. This typically models the uncertainty deriving from shot noise.

The FFT of \( u^2[h(q, p)] \) is, in this case,

\[ \hat{U}_s(n, m) = k \hat{H}(n, m) + \text{FFT}(u_0^2) \]
\[ = \begin{cases} k NM \langle h \rangle + NM u_0^2 & \text{if } n = m = 0 \\ k \hat{H}(n, m) & \text{otherwise} \end{cases}. \]
\[ \text{(31)} \]

with

\[ \langle h \rangle = \frac{1}{NM} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} h(q, p) = \frac{\hat{H}(0,0)}{NM}. \]
\[ \text{(32)} \]

Equation (31) can be substituted into Eqs. (13)–(15), and (29) resulting in

\[ u^2(\text{Re} \tilde{H}) = \begin{cases} NM \langle k \rangle + u_0^2 & \text{if } n = m = 0 \\ \frac{1}{4} NM \left\{ k \langle h \rangle + \frac{\text{Re} \tilde{H}_2(2n, 2m)}{NM} \right\} + u_0^2 & \text{otherwise} \end{cases}. \]
\[ \text{(33)} \]
\[
\begin{align*}
\sigma^2(\text{Im } \hat{H}) &= \begin{cases} 
0 & \text{if } n = m = 0 \\
\frac{1}{2} \text{NM} \left\{ k \left[ \langle h \rangle - \frac{\text{Re} \hat{H}(2n, 2m)}{\text{NM}} \right] + u_0^2 \right\} & \text{otherwise}
\end{cases}, \\
\sigma^2(\phi_{12}) &= \begin{cases} 
0 & \text{if } n = m = 0 \\
\frac{1}{2} \left[ \frac{1}{|H_1|^2} \text{NM}(k \langle h_1 \rangle + u_0^2) + \frac{1}{|H_2|^2} \text{NM}(k \langle h_2 \rangle + u_0^2) + \frac{(\text{Im } \hat{H}_1)^2 - (\text{Re } \hat{H}_1)^2}{|H_1|^2} k \text{ Re } \hat{H}_1(2n, 2m) \\
+ \frac{(\text{Im } \hat{H}_2)^2 - (\text{Re } \hat{H}_2)^2}{|H_2|^2} k \text{ Re } \hat{H}_2(2n, 2m) - 2 \text{ Re } \hat{H}_1 \text{ Im } \hat{H}_1 k \text{ Im } \hat{H}_1(2n, 2m) - 2 \text{ Re } \hat{H}_2 \text{ Im } \hat{H}_2 k \text{ Im } \hat{H}_2(2n, 2m) \right] & \text{otherwise}
\end{cases}
\end{align*}
\]

Here, again, \( \phi_{12} = \phi_{12}(n, m) \) and \( \hat{H}_i = \hat{H}_i(n, m) \) are local values while \( k \), \( u_0^2 \), and \( \langle h_i \rangle \) are global, hologram-wide, pixel-independent values.

### 2.3.1 Simplification for holograms of speckle patterns

In the FFT of an hologram generated by the interference of a speckle pattern with an uniform (or nearly uniform) reference beam, typically,

\[
\begin{align*}
\text{Re } \hat{H}(2n, 2m) &\approx \hat{H}(0,0) = \text{NM}(\langle h \rangle) \\
\text{Im } \hat{H}(2n, 2m) &\approx \hat{H}(0,0) = \text{NM}(\langle h \rangle)
\end{align*}
\]

for \( n \neq 0 \) and \( m \neq 0 \),

and Eqs. (33)–(36) can be approximated as

\[
\begin{align*}
\sigma^2[\text{Re } \hat{H}(n, m)] &= \begin{cases} 
\text{NM}(k \langle h \rangle + u_0^2) & \text{if } n = m = 0 \\
\frac{1}{2} \text{NM}(k \langle h \rangle + u_0^2) & \text{otherwise}
\end{cases}, \\
\sigma^2[\text{Im } \hat{H}(n, m)] &= \begin{cases} 
0 & \text{if } n = m = 0 \\
\frac{1}{2} \text{NM}(k \langle h \rangle + u_0^2) & \text{otherwise}
\end{cases}, \\
\sigma[\text{Re } \hat{H}(n, m), \text{Im } \hat{H}(n, m)] &= \begin{cases} 
0 & \text{if } n = m = 0 \\
\frac{1}{2} k \text{ Im } \hat{H}(2n, 2m) & \text{otherwise}
\end{cases}, \\
\sigma^2(\phi_{12}(n, m)) &= \begin{cases} 
0 & \text{if } n = m = 0 \\
\approx k \xi(n, m) + u_0^2 \eta(n, m) & \text{otherwise}
\end{cases}
\end{align*}
\]

with

\[
\xi = \xi(n, m) = \frac{\text{NM}}{2} \left( \frac{\langle h_1 \rangle}{|H_1(n, m)|^2} + \frac{\langle h_2 \rangle}{|H_2(n, m)|^2} \right),
\]

\[
\eta = \eta(n, m) = \frac{\text{NM}}{2} \left( \frac{1}{|H_1(n, m)|^2} + \frac{1}{|H_2(n, m)|^2} \right).
\]

If, in addition, the illumination conditions and the object are the same for both of the holograms used to calculate the phase change, it will be reasonable to assume that

\[
\langle h_1 \rangle \approx \langle h_2 \rangle \approx \langle h_{12} \rangle \approx \frac{\langle h_1 \rangle + \langle h_2 \rangle}{2} = \langle h_{12} \rangle, \\
\xi \approx \langle h_{12} \rangle \frac{\text{NM}}{2} \left( \frac{1}{|H_1|^2} + \frac{1}{|H_2|^2} \right) = \langle h_{12} \rangle \eta,
\]

and the expression of the squared standard uncertainty, Eq. (41), is further simplified to

\[
\sigma^2(\phi_{12}(n, m)) \approx (k \langle h_{12} \rangle + u_0^2) \eta(n, m) \quad \text{otherwise}.
\]

### 3 Experiment

To verify the validity of the estimations of the uncertainty yielded by the expressions derived in Sec. 2.3, we have conducted a set of experimental recordings of Fourier transform digital holograms of optically rough objects, subsequent complex-amplitude reconstructions and phase-change measurements, and eventually compared the actual observed values of the corresponding sample variances, which constitute a type A evaluation of the squared standard uncertainty of the measurements, with the type B estimation of the square of the standard uncertainty provided by Eqs. (38), (39), and (41).

The experiments have been arranged to record pairs of nominally identical holograms and, therefore, to get nominally identical reconstructed complex amplitude and null phase change for all of the pixels. Since, according to Eqs. (38) and (39), the uncertainties of the real and imaginary parts of the complex amplitude should be the same for all of the pixels—excluding the one at (0, 0)—the corresponding variances can be calculated by comparing pixel by pixel the reconstructed fields corresponding to the two holograms in each pair. On the other hand, the variance of the phase change corresponding to given values of \( \xi \) and \( \eta \) in Eq. (41) can be calculated by comparing the phase-change values corresponding to the pixels with such values of \( \xi \) and \( \eta \) with the average phase change in the whole of the object, which should not be significantly different from zero.

The effect of the average value of the digital holograms \( \langle h \rangle \)—i.e., of the hologram’s illumination level—on the
uncertainty has been analyzed by recording pairs of holograms with 150 different values of the exposure time ranging from 0.02 ms to 3.00 ms in 0.02 ms steps which, for the experimental system described in Sec. 3.1, covers from near the camera noise threshold to saturation in at most 0.25% of the hologram pixels. The measurements have been repeated with 24 different objects to take into account a reasonable number of independent speckle patterns.

### 3.1 Experimental Arrangement

The holograms have been acquired with a hybrid lensless Fourier transform digital holographic camera, which has been fully described in Ref. 14. As shown in Fig. 1(a), the object is illuminated with a continuous-emission frequency-doubled Nd:YAG laser and its image is projected with an objective lens on a plane, where a rectangular aperture limits the extension of the object field. A lensless Fourier transform hologram is eventually generated by adding a fiber-optic guided reference beam diverging from this plane. The size of the aperture and its position relative to the sensor and to the reference-beam source are carefully chosen to get a critically sampled hologram as well as to prevent the overlapping of the object image and the autocorrelation terms in the subsequent reconstruction, as shown in Fig. 1(b).

The holograms are recorded as 2452 × 2054 pixels, 14 bit deep, images using an Allied Vision Technologies model Pike F-505-B camera, equipped with the SONY ICX625A CCD sensor. To minimize the effects of air convection and thermal instability in the reference-beam optical fiber, the two holograms in each experiment are acquired with a delay of 70.4 ms, which is very near to the minimum allowed by the camera.

The test objects are 24 nonoverlapping regions, with an area of 128 mm × 32 mm each, from the surface of three different 250 mm × 250 mm × 10 mm uncoated aluminum plates. The plate under test is fixed to the same table as the optical system and regarded rigid enough to assume that the phase difference due to its displacement is nominally $\phi_{12} = 0$.

### 3.2 Values of the Linear Uncertainty Model Parameters

The squared uncertainty of the digital holograms $u^2[h(q, p)]$ can be identified with the expected value of the temporal variance $\sigma^2$ of the digital signal that the camera outputs which, according to the noise model adopted for the EMVA 1288 standard, can be written as

$$\sigma^2 = k^2 \sigma^2_d + \sigma_q^2 + k(\mu_y - \mu_{y,\text{dark}}).$$

(47)

with

$$k^2 \sigma^2_d = k^2 \sigma^2_{d,0} + k^2 \mu_{t,\text{dark}} t_{\text{exp}} = k^2 \sigma^2_{d,0} + k \mu_{t,\text{dark}} t_{\text{exp}},$$

(48)

$$\mu_{y,\text{dark}} = k \mu_d = k \mu_{d,0} + k \mu_{t,\text{dark}} t_{\text{exp}} = k \mu_{d,0} + \mu_{t,\text{dark}} t_{\text{exp}},$$

(49)

where $\mu_y$ is the expected value of the output signal $y$ of the camera expressed in DN (i.e.: “data number” or “counts”), $k$ is the overall system gain, $\sigma^2_d$ is the variance of the dark noise (without the effects of quantization), $\sigma^2_q$ is its value when the exposure time approaches zero, $\sigma^2_{d,0}$ is the variance of the quantization noise expressed in DN$^2$, $\mu_{t,\text{dark}}$ is the expected value of the dark current, $t_{\text{exp}}$ is the exposure time, $\mu_d$ is the expected value of the dark signal, and $\mu_{d,0}$ is its value when the exposure time approaches zero. The subindex “d” relates to dark signal expressed in e$^-$ (electrons) and the subindex “dark” to dark signal expressed in DN.

Substituting Eqs. (48) and (49) into Eq. (47) results in

$$\sigma^2 = k^2 \sigma^2_{d,0} + k \mu_{t,\text{dark}} t_{\text{exp}} + \sigma_q^2 + k \mu_d - k \mu_{d,0} - k \mu_{t,\text{dark}} t_{\text{exp}}$$

$$= k \mu_d + \left[(k^2 \sigma^2_{d,0} + \sigma_q^2) - k \mu_{t,\text{dark}} t_{\text{exp}} \right]$$

$$= k \mu_d + (\sigma^2_{y,\text{dark,0}} - k \mu_{t,\text{dark,0}}).$$

(50)

where $\sigma^2_{y,\text{dark,0}}$ is the value of the variance of the dark noise (including the effects of quantization and expressed in DN) when the exposure time approaches zero.

The expected value of the signal $\mu_y$ in Eq. (50) can be identified with the local value of the hologram $h(q, p)$ and, thus, the values of the parameters in Eq. (30) are the overall system gain of the camera, $k$, and

$$u^2_0 = \sigma^2_{y,\text{dark,0}} - k \mu_{t,\text{dark,0}}.$$ 

(51)

The gain and temporal dark noise characteristics of the camera were measured by the photon transfer method,$^{15}$ following the procedures specified in Secs. 6.3 (method I), 6.6
and 7.1 of the EMVA 1288 standard. The resulting values are listed in Table 1, and the corresponding values of the uncertainty parameters are

\[ k = 3.072 \, \text{DN/e}^-, \]

\[ u_0^2 = 1673 \, \text{DN}^2. \]

### 3.3 Data Processing

The following procedure was applied to analyze the degree of agreement between the sample variances resulting from the measurements—\(s^2(\text{Re} \, \hat{H})\), \(s^2(\text{Im} \, \hat{H})\), and \(s^2(\phi_{12})\)—and the corresponding values of the squared standard uncertainty yielded by Eqs. (38), (39), and (41).

1. For each of the 7200 individual digital holograms recorded as described in Sec. 3.1:
   a. Calculate \(\langle h \rangle\) according to Eq. (32).
   b. Reconstruct the complex-amplitude field of the image of the object \(\hat{H}\) by using Eq. (3); i.e., we assume that \(\Delta x = \Delta y = 1\). The region corresponding to the reconstructed image of the object has \(N \times M' = 2452 \times 513\) pixels with \(0 \notin [m_{\text{min}}, m_{\text{max}}]\), as shown in Figs. 1(b) and 1(c).

2. For the two reconstructed fields corresponding to each of the 3600 pairs of nominally identical holograms:
   a. Calculate the phase-change map by using Stetson and Brohinsky’s algorithm according to Eq. (24) [see Fig. 1(d)].
   b. Calculate the maps of \(\xi(n, m)\), \(\eta(n, m)\), and of the corresponding local squared standard uncertainty \(u^2(n, m)\) by using Eqs. (42), (43) and (41), respectively, with the values of \(k\) and \(u_0^2\) given in Eqs. (52) and (53). [Note that in Eqs. (42) and (43) \(M = 2045\) pixels, the full height of the holograms.]
   c. Calculate the weighted average of the phase change as
      \[ \langle \phi_{12} \rangle = \frac{\sum_{n=0}^{N-1} \sum_{m=0}^{m_{\text{max}}} \phi_{12}(n, m) / \sigma^2(n, m)}{\sum_{n=0}^{N-1} \sum_{m=0}^{m_{\text{max}}} \sigma^2(n, m)}. \]
   d. Discard the hologram pair if \(\langle \phi_{12} \rangle \geq 0.05\) rad, i.e., if the equivalent average plate displacement is greater than approximately 2 nm. Such large average phase-changes would be associated with changes of the real or imaginary part of the reconstructed complex field—induced by seismic noise, air turbulence, laser instability, and so on—which may be greater than 5% of its modulus at some of the pixels and, therefore, we assume that in these conditions, the two holograms in the pair depart significantly from identity.

After this step, 3070 hologram pairs remained valid.

3. For each of the remaining pairs of nominally identical complex-amplitude holographic reconstructions, calculate the average variances of the real and imaginary parts as

\[ s^2(\text{Re} \, \hat{H}) = \frac{1}{2(NM' - 1)} \sum_{n=0}^{N-1} \sum_{m=m_{\text{min}}}^{m_{\text{max}}} [\text{Re} \, \hat{H}_1(n, m) - \text{Re} \, \hat{H}_2(n, m)]^2, \]

\[ s^2(\text{Im} \, \hat{H}) = \frac{1}{2(NM' - 1)} \sum_{n=0}^{N-1} \sum_{m=m_{\text{min}}}^{m_{\text{max}}} [\text{Im} \, \hat{H}_1(n, m) - \text{Im} \, \hat{H}_2(n, m)]^2. \]

4. Step 3 together with 1a result in two sets of corresponding values of \(\langle h_{12} \rangle\)—as in Eq. (44)—and of \(s^2(\text{Re} \, \hat{H})\) and \(s^2(\text{Im} \, \hat{H})\), respectively. These experimental values of the variance are identified to the corresponding expressions of the squared standard uncertainty, Eqs. (38) and (39):

\[ s^2(\text{Re} \, \hat{H}) = \frac{1}{2} NM(k_R \langle h_{12} \rangle + u_0^2 R) \]

\[ s^2(\text{Im} \, \hat{H}) = \frac{1}{2} NM(k_I \langle h_{12} \rangle + u_0^2 I), \]

and the values of \(k\) and \(u_0^2\) that optimize the agreement between the measurements and the uncertainty model are fitted with the Marquardt–Levenberg (M-L) method, taking \(s^2(\text{Re} \, \hat{H})\) and \(s^2(\text{Im} \, \hat{H})\) as the uncertainties of the respective estimations of the variances, which the fitting algorithm uses to weight the data. We used for this task the implementation of the M-L method in the “gnuplot”

### Table 1 Noise parameters of the Pike F-505-B camera, measured by the photon transfer method.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Measured value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall system gain</td>
<td>(k)</td>
<td>3.072</td>
<td>DN/e^-</td>
</tr>
<tr>
<td>Temporal dark noise</td>
<td>(\phi_{\text{dark},0})</td>
<td>1692</td>
<td>DN^2</td>
</tr>
<tr>
<td>Dark signal</td>
<td>(h_{\text{dark},0})</td>
<td>6.03</td>
<td>DN</td>
</tr>
<tr>
<td>Dark current</td>
<td>(\mu_{\text{dark}})</td>
<td>-11</td>
<td>DN/s</td>
</tr>
</tbody>
</table>

The sets of measured points as well as the lines corresponding to the expected squared standard uncertainty—with the values of the parameters resulting from the characterization of the camera, Eqs. (52) and (53)—and to the M-L fit are shown in Fig. 2. The relative differences between the measured variances and the expected squared standard uncertainty, as well as between the variances of the real and imaginary parts of the reconstructions, are plotted in Fig. 3.
5. For each of the 3070 phase-change maps remaining after step 2:

   a. Classify its $NM^\prime = 2452 \times 513$ pixels into $50 \times 50$ sets according to the corresponding values of $\xi$ and $\eta$ for the ranges $0 \leq k_\xi \leq (\pi/3)^2$ and $0 \leq u_{0\eta}^2 \leq (\pi/3)^2$, with the values of $k$ and $u_{0\eta}^2$ in Eqs. (52) and (53).

   b. Discard the sets with less than 50 pixels as nonstatistically significant.

   c. For each of the remaining sets of pixels, calculate the variance $s^2_{\xi,\eta}(\phi_{12})$ of the phase change of the pixels in the set, taking the weighted average of the corresponding phase-map $\langle \phi_{12} \rangle$ obtained in step 2c as the expected value of the phase change.

6. The resulting $3070 \times 50 \times 50$ experimental values of the variance are identified to the corresponding expression of the squared standard uncertainty, Eq. (41):

$$s^2_{\xi,\eta}(\phi_{12}) = k_{\phi} \xi + u_{0\phi}^2 \eta,$$

and the values of $k$ and $u_{0\phi}^2$ are fitted by using the M-L method, taking $s^2_{\xi,\eta}(\phi_{12})/2/(p_{\xi,\eta} - 1)$ as the uncertainty of the variance, with $p_{\xi,\eta}$ the number of pixels in the set used to obtain the corresponding value.

Figure 4(a) shows a 3-D scatter diagram of the experimental values of the variance of the phase change and the plane of expected squared standard uncertainty, Eq. (41) with the values in Eqs. (52) and (53). The relative differences between the measurements and the corresponding points of the expected plane are scatter plotted in Fig. 4(b). Due to the large number of experimental values, only one of every eleven is plotted in the figure to reduce cluttering.

7. For each of the $P = 3070$ phase-change maps remaining after step 2:

   a. Classify the pixels according to the value of the expected uncertainty of the phase change into 50 sets, equally spaced in the range $0 \leq u(\phi_{12}) \leq \pi/3$. Discard the pixels out of the range.

   b. Discard the sets with less than 50 pixels.

   c. For each of the remaining sets of pixels, calculate the variance of the corresponding phase change $s^2_{u}(\phi_{12})$ taking, once again, $\langle \phi_{12} \rangle$ as the expected value.
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Fig. 4 (a) Variance of the phase change of the pixels with given values of \( \xi \) and \( \eta \), Eqs. (42) and (43), for each of the analyzed phase maps and plane of expected squared standard uncertainty, Eqs. (41) with (52) and (53). (b) Relative difference between the measured variance and the expected squared standard uncertainty.

8. For each of the 50 sets, corresponding to a given subrange of values of the expected uncertainty, combine the variances obtained for the \( P \) phase-change maps according to the number of pixels \( n_{u,i} \) in the corresponding set:

\[
\sigma_u^2(\phi_{12}) = \frac{\sum_{i=1}^{P} n_{u,i} - 1}{\sum_{i=1}^{P} n_{u,i} - 1} \sum_{i=1}^{P} \sigma_u^2(\phi_{12}).
\] (60)

The resulting values of the experimental standard deviation (type A evaluation of the uncertainty) are plotted against the expected uncertainty (type B evaluation) in Fig. 5. The cumulative pixel frequency corresponding to each subrange of the uncertainty is also plotted, relative to the total number of analyzed pixels including those out of range and in discarded sets.

4 Discussion

4.1 Uncertainty of the Complex Amplitude of the Reconstructed Field

The measurements presented in Fig. 2 show a good agreement between the actual variances of both the imaginary and real parts of the reconstructed complex-amplitude field and the value of the squared standard uncertainty estimated as a function of the corresponding average value of the digital holograms. The values of the parameters of the uncertainty model \((k_R, u^2_{0,R}, k_I, u^2_{0,I})\), which best fit Eqs. (38) and (39) to the actual measurements (Table 2), are reasonably close to the reference values \( k \) and \( u^2_0 \) obtained through the characterization of the digital camera (Sec. 3.2).

It is remarkable that most of the measured values of the variance are greater than the expected squared standard uncertainty. This becomes apparent by looking at the relative difference between the measured variances and the expected squared standard uncertainty, represented in Figs. 3(a) and 3(b). Most of the measured points concentrate above, but not far—less than 5% away—from the expected value. This can be attributed to the presence of uncertainty sources other than the camera noise (e.g., seismic noise, air turbulence, laser instability, and so on), though of moderate magnitude. As the average hologram value \( \langle h \rangle \) approaches zero, the measured variances become slightly lower than the squared standard uncertainty. Such behavior may be related to the saturation-to-zero at the lowest sensor illumination levels, which would hide part of the camera noise; this hypothesis may be tested in the future by repeating the measurements with increasing values of the camera amplifier’s offset. However, the variances of the real and imaginary parts of the reconstructed field are essentially identical even for low average hologram values, Fig. 3(c), in accordance with the equal values of the squared standard uncertainty estimated with Eqs. (38) and (39).

4.2 Uncertainty of the Phase-Change Measurements

There is a reasonable agreement (see Fig. 4) between the variance of the measured phase change, type A evaluated for sets of points with given values of \( \xi \) [Eq. (42)] and \( \eta \) [Eq. (43)], and the corresponding values of the squared standard uncertainty, type B evaluated by using Eq. (41). This agreement is, however, not as good as the aforementioned for the real and imaginary parts of the reconstructed complex amplitude.

The corresponding best fit value of the overall camera gain \( k_\phi \) (Table 2) is comparable to those obtained for the complex amplitude \((k_R \) and \( k_I)\), but the relative difference between \( u^2_{0,\phi} \) and its reference value \( u^2_0 \) is four times larger. This may be related with the fact that, for the smallest values of \( \xi \) and \( \eta \), some estimations of low squared standard uncertainty are grossly mismatched with the corresponding values.

Fig. 5 Actual standard deviation of phase change (type A evaluation) of the pixels with a given expected uncertainty (type B evaluation), estimated by using Eqs. (41) with (52) and (53), for the whole set of 3070 experimental phase maps. The cumulative pixel frequency is relative to a total of 3.867 \( \times 10^9 \) pixels.
of the experimental variance, as can be observed pointed by an arrow close to the $s_{\Delta \phi}^2(\eta)$ axis in Fig. 4(a). A plausible cause of this behavior is the presence of spatially periodical electrical noise in the camera output, which, for the holograms with lower average values $\langle h \rangle$, results in local peaks of spectral power much higher than the optical signal. This noise is readily apparent as bright spots when such low-valued holograms are digitally reconstructed, as shown in Fig. 6.

The comparison of the actual standard deviation of the phase-change measurements with the standard uncertainty yielded by Eq. (41) [see Fig. 5] reveals a very good agreement for values of the standard uncertainty below approximately 0.5 rad, which include ~75% of the analyzed pixels.

Larger values of the uncertainty are clearly underestimated. Since a first order approximation was used for the propagation of uncertainty in phase-change calculation (Sec. 2.2), the observed deviation can be a consequence of the nonlinearity of the arc tangent function. On the other hand, the lowest values of the uncertainty are slightly lower than the corresponding experimental standard deviation values. This is most likely due to the apparent low standard uncertainty associated to the large value of the reconstructed amplitude in the pixels affected by the aforementioned periodical electrical noise.

### 5 Conclusions

We have derived general expressions, which allow type B evaluation of the local standard uncertainties of the real and imaginary parts of the reconstructed complex-amplitude fields in Fourier- and quasi-Fourier transform digital holographic interferometry [Eqs. (13)–(15)] as well as of the standard uncertainty of the measurements of the phase change between two holographic reconstructions [Eq. (29)]. These expressions are increasingly simplified by assuming a linear behavior of the sources of squared uncertainty in hologram recording [Eqs. (33)–(36)], that the object beam is a speckle pattern [Eqs. (38)–(41)] and eventually, for the uncertainty of the phase change, the same average value in both holograms [Eq. (46)].

The phase-change or complex-amplitude maps together with the corresponding standard uncertainty maps, as those provided by these expressions, constitute a metrologically meaningful output for a digital holographic measurement system.

The correspondence between the type B evaluation of the squared standard uncertainties provided by Eqs. (38), (39) and (41) and the actual, type A evaluated, sample variances of the measured real and imaginary parts of the complex amplitude and phase change, respectively, has been tested through an extensive set of experiments conducted under repeatability conditions. The parameters of the hologram uncertainty model have been derived from the noise characteristics of the camera used to record the digital holograms, which were measured according to the EMVA 1288 standard.

A good agreement has been found between the estimations of the squared standard uncertainties of the real and imaginary parts of the complex amplitude of the holographic reconstructions and the corresponding experimental variances. A moderate discrepancy has been observed for holograms with very low average values; this is probably related to effects of saturation-to-zero and will require further investigation.

The estimations of the standard uncertainty of the phase change between pairs of holograms match remarkably well the corresponding experimental standard deviations for values of the standard uncertainty below approximately 0.5 rad. This is an acceptable range since phase values returned by the arc tangent function lie in the interval $[-\pi \text{ rad}, +\pi \text{ rad}]$. For higher values, however, the measured standard deviations are greater than the estimations of the uncertainty. The possible connection of this behavior and the nonlinearity of the arc tangent function may be tested in the future by propagating the uncertainty using a Monte-Carlo method. The presence of periodic electrical noise in digital holograms with low average value leads, apparently, to a
References


Ángel F. Doval received his automatics and electronics engineer and PhD degrees from the Universities of Santiago de Compostela in 1990 and Vigo in 1997, respectively. He joined the Department of Applied Physics of the University of Vigo in 1998 as an assistant professor and has been associate professor since 1998. His research with the Optical Metrology Group is focused on optical measurement and nondestructive inspection with ESPI, digital holography, and other interferometric techniques.

Cristina Trillo received her automatics and electronics engineer and PhD degrees from the University of Vigo in 1999 and 2004, respectively. She joined as a research associate in the Department of Applied Physics of the same university in 2000, where she is currently an associate professor. Her research interests with the Optical Metrology Group include phase evaluation techniques, electronic speckle pattern interferometry, and digital holography.

J. Carlos López-Vázquez received his MPhil degree in physics from the University of Santiago de Compostela in 1988 and his PhD degree in applied physics from the University of Vigo in 1997, where he is currently an associate professor. His research interests include optical measurement techniques, nondestructive inspection techniques and modeling of wave propagation.

José L. Fernández received his diploma in mechanical engineering from Universidad Politécnica de Madrid in 1984 and his PhD degree in engineering in 1988 from Universidad de Santiago de Compostela. He is currently a full professor of applied physics and head of the Optical Metrology Group at the University of Vigo. His research interests include optical measurements and nondestructive inspection techniques, specifically TV holography and interferometry.