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Cooperative games and cost allocation problems

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Abstract The objective of this paper is to provide a general view of the literature of applications of transferable utility cooperative games to cost allocation problems. This literature is so large that we concentrate on some relevant contributions in three specific areas: transportation, natural resources and power industry. We stress those applications dealing with costs and with problems arisen outside the academic world.

Keywords Cooperative TU games · cost allocation problems · game practice

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1 Introduction

Game theory is a mathematical theory which deals with interactive decision situations. It approaches those situations making use of both non-cooperative and cooperative models. The differences between these two kind of models are explained in van Damme and Furth (2002). They write that "non-cooperative models assume that all the possibilities for cooperation have been included as formal moves in the game, while cooperative models are 'incomplete' and allow players to act outside of the detailed rules that have been specified". In other words, when the cooperation mechanisms are too complex as to be fully described by a mathematical model we enter the terrain of cooperative game theory. It does not deal with strategic analysis, but with coalitions and allocations and, as indicated in González-Díaz et al (2010), it "considers groups of players willing to allocate the joint benefits derived from their cooperation (however it takes place)". The most important models within cooperative game theory are the so-called transferable utility games. They assume that the benefits generated by a coalition of players can be freely distributed among the players. From now on, when we write cooperative games we mean transferable utility games.

Cooperative games have many applications. For instance, they have been widely used to analyse voting institutions (see Laruelle and Valenciano (2008)), they are a helpful tool for allocating benefits when an operational research problem can improve its performance if several agents cooperate (see, for example, Borm et al (2001)), or they can be a good instrument to propose solutions for bankruptcy situations (see Thomson (2003)). Other applications of cooperative games in several fields can be found in Moretti and Patrone (2008).

In this paper, we concentrate on the applications of cooperative games to cost allocation problems. In a cost allocation problem there is a group of agents that are willing to perform a joint project. The project has some ingredients which are common for all the agents but it also has some specific elements which are not common. The main issue in a cost allocation problem is to allocate the cost derived from the joint performance of the project incorporating all the special features specified by the agents. Notice that there are not essential differences between allocating costs or allocating benefits, because a cost allocation problem gives rise to a benefits allocation problem[1] and vice versa. However, there are formal and interpretation differences; this paper is written from the perspective of costs and we mainly deal with problems arising in a cost environment.

The objective of this paper is to provide a general view of the literature of applications of cooperative games to cost allocation problems. This literature is so large that we concentrate on some relevant contributions in three specific areas, which at the end cover a large part of this field: transportation, natural

[1] The benefit of a group is the sum of the individual costs minus the cost of the whole group.

resources and power industry. We stress those applications dealing with costs and with problems arisen outside the academic world. In this sense we are interested more in game practice than simply in applied game theory. Like in Patrone et al (2000) we mean by game practice those applications of game theory that address real problems faced by real decision-makers outside the academic world. There are other papers on cooperative games and cost allocation, like Tijs and Driessen (1986) and Young (1994). However, those papers emphasize the theoretical issues, whereas here we put more emphasis on the examples and on the game practical aspects. Lemaire (1984) is another work on cost allocation which is more concerned with examples, but it is clear that the field has advanced much in the last decades.

The organization of the paper is as follows. In Section 2 we provide a concise introduction to game theoretic methods in cost allocation problems. The objective of that section is to fix the notation that we use later on, and also to help those readers who are not familiar with the topic. In Sections 3, 4 and 5 we review some applications of cooperative games to cost allocation problems in the areas of transportation, natural resources and power industry, respectively. In each of those sections we tackle one representative problem and explain it more carefully. To finish the paper we include a section of concluding remarks.

2 Cost allocation problems

In this section we give some basic definitions and results regarding cost allocation problems. As we have written above, in a cost allocation problem there is a group of agents that are willing to perform a joint project and to allocate its cost among them. Typically, each agent needs that some special features are included in the project, although its main characteristics are common to all agents. Thus, to describe a cost allocation problem we have to indicate the set of agents involved and, for each possible group of agents, the cost of the project with the specifications indicated by the agents in that group. We give below the mathematical definition of a cost allocation problem.

Definition 1 A cost allocation problem is a pair (N, c) such that:

- N is the finite set of agents who are potential users of the project, and
- $c : 2^N \to \mathbb{R}$ is a map which assigns to every possible group of agents $S \subset N$ the cost of the project with the specifications needed by the members of S, being $c(\emptyset) = 0$. The objective is to allocate $c(N)$ among the agents in N. In general we identify (N, c) with c. For every finite set N we denote by $\mathcal{C}(N)$ the set of cost allocation problems with set of agents N. We denote by $\mathcal{C}$ the set of cost allocation problems with a finite set of agents.

Notice that a cost allocation problem presented in this way is the same as a transferable utility game, although cost allocation problems deal with costs instead of with benefits and the interpretations of concepts and results in this context are in some sense dual to the interpretations of the corresponding concepts and results for transferable utility games. Before going on let us see a toy example to illustrate the above definition.

Example 1 Three research groups belonging to the universities of Milano, Italy (group one), Genova, Italy (group two) and Vigo, Spain (group three) plan to invite a Japanese professor to give a course on game theory. To minimize the cost, they coordinate the courses, so that the professor makes a tour visiting Milano, Genova and Vigo. The groups want to allocate the cost of the tour among them. For that purpose they have estimated the travel cost (in euros) of the visit for all the possible coalitions of groups: $c(1) = 1500$, $c(2) = 1600$, $c(3) = 1900$, $c(12) = 1600$, $c(13) = 2900$, $c(23) = 3000$, $c(N) = 3000$ (for every S, $c(S)$ indicates the cost of the tour that the professor should make to visit all the groups in S)[2].

In general, we deal with cost allocation problems whose agents are really interested in cooperation, in the sense that for every two disjoint groups of agents the cost of the project if they merge is smaller than or equal to the costs of the projects they would perform separately. This is what we call a subadditive cost allocation problem. Next we give the formal definition of subadditivity and introduce a condition which is stronger than subadditivity: the concavity condition.

Definition 2 – We say that a cost allocation problem $c \in \mathcal{C}(N)$ is subadditive if, for all $S, T \subset N$ with $S \cap T = \emptyset$, it holds that $c(S \cup T) \leq c(S) + c(T)$.

[2] To avoid cumbersome notations, when no confusion is possible, we write $c(1)$, $c(12)$, ..., instead of the more formal $c(\{1\})$, $c(\{1,2\})$, ...

– We say that a cost allocation problem $c \in \mathcal{C}(N)$ is concave if, for all $S, T \subset N$, it holds that $c(S \cup T) + c(S \cap T) \leq c(S) + c(T)$. Equivalently (see Driessen (1988)), c is concave if, for each $i \in N$ and each pair $S, T \subset N \setminus \{i\}$ with $S \subset T$, it holds that $c(S \cup \{i\}) - c(S) \geq c(T \cup \{i\}) - c(T)$.

It has been said above that the objective when dealing with a cost allocation problem $c \in \mathcal{C}(N)$ is to allocate $c(N)$ among the agents in N. To that aim, what we really want is to define sensible allocation rules for cost allocation problems. We give now the formal definition of an allocation rule.

Definition 3 An allocation rule for a class of cost allocation problems $\mathcal{A} \subset \mathcal{C}$ is a map ϕ which assigns to every $c \in \mathcal{C}(N) \subset \mathcal{A}$ an element $\phi(c) \in \mathbb{R}^N$ satisfying that $\sum_{i \in N} \phi_i(c) = c(N)$. When $\mathcal{A} = \mathcal{C}$ we simply say that ϕ is an allocation rule for cost allocation problems.

One of the most important allocation rules for cost allocation problems taken from game theory is the Shapley value, introduced in Shapley (1953). Next we introduce it axiomatically, i.e. we give some reasonable properties for an allocation rule and prove that there is only one rule which satisfies those properties; that rule is the Shapley value.

Definition 4 Let $c \in \mathcal{C}(N)$ be a cost allocation problem.

– $i, j \in N$ are said to be symmetric in c if, for every group $S \subset N \setminus \{i, j\}$, it holds that $c(S \cup \{i\}) = c(S \cup \{j\})$.
– $i \in N$ is said to be a null agent in c if, for every group $S \subset N$, it holds that $c(S \cup \{i\}) = c(S)$.

Symmetry (SYM). An allocation rule ϕ satisfies SYM if, for all $c \in \mathcal{C}(N)$ and all $i, j \in N$ which are symmetric in c, it holds that $\phi_i(c) = \phi_j(c)$.

Null Agent Property (NU). ϕ satisfies NU if, for all $c \in \mathcal{C}(N)$ and all null agent $i \in N$ in c, $\phi_i(c) = 0$.

Additivity (ADD). ϕ satisfies ADD if, for all $c, d \in \mathcal{C}(N)$, it holds that $\phi(c + d) = \phi(c) + \phi(d)$.

Theorem 1 *There exists a unique allocation rule for cost allocation problems satisfying SYM, NU and ADD. This rule is called the Shapley value and maps every $c \in \mathcal{C}(N)$ to $\Phi(c) \in \mathbb{R}^N$ given by:*

$$\Phi_i(c) = \sum_{S \subset N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} (c(S \cup \{i\}) - c(S)),$$

for all $i \in N$, where $s = |S|$ and $n = |N|$. $\Phi(c)$ is said to be the Shapley value of c.

When a cost allocation rule is proposed, it is important to know if it provides stable allocations. The concept which is more commonly used to embody stability is the core, that we introduce below.

Definition 5 Let $c \in \mathcal{C}(N)$ be a cost allocation problem.

– The set of imputations of c is given by:

$$I(c) = \{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = c(N) \text{ and, moreover, } x_i \leq c(i) \ \forall i \in N\}.$$

The elements of $I(c)$ are called imputations of c.

– The core of c is given by:

$$Co(c) = \{x \in I(c) \mid \sum_{i \in S} x_i \leq c(S) \text{ for all } S \subset N\}.$$

The elements of $Co(c)$ are called core allocations of c.

It is easy to check that a cost allocation problem may have an empty core. For those problems having non-empty cores, it would be very convenient to allocate the total cost through a core allocation, because only core allocations guarantee that no group of agents is disappointed. However, stability (as embodied in the core concept) and fairness (as embodied in the Shapley value) are not always compatible. For instance, the Shapley value of a cost allocation problem may lie outside its core, even when the later is non-empty. The next result (whose proof can be seen, for instance, in González-Díaz et al (2010)) relates the Shapley value with the core.

Theorem 2 *Take $c \in \mathcal{C}$.*

1. *If c is subadditive, then $\Phi(c) \in I(c)$.*
2. *If c is concave, then $\Phi(c) \in Co(c)$.*

Let us illustrate these concepts and results in the problem of Example 1.

Example 2 Consider the visiting professor problem of Example 1: $c(1) = 1500$, $c(2) = 1600$, $c(3) = 1900$, $c(12) = 1600$, $c(13) = 2900$, $c(23) = 3000$, $c(N) = 3000$. Then,

$$I(c) = \{x \in \mathbb{R}^3 \mid x_1 \leq 1500, x_2 \leq 1600, x_3 \leq 1900, x_1 + x_2 + x_3 = 3000\},$$

and

$$Co(c) = \{x \in I(c) \mid x_1 + x_2 \leq 1600, x_1 + x_3 \leq 2900, x_2 + x_3 \leq 3000\}.$$

It is easy to check that $Co(c)$ is the convex hull of the set

$$\{(1000, 100, 1900), (0, 1100, 1900), (0, 1600, 1400), (1500, 100, 1400)\},$$

and that $\Phi(c) = (2000/3, 2300/3, 4700/3)$. Notice that $\Phi(c) \in Co(c)$; it can be checked directly, or using Theorem 2 and the fact that c is concave.

Another important allocation rule for cost allocation problems is the nucleolus, introduced in Schmeidler (1969). Whereas the Shapley value is essentially linked to fairness, the nucleolus is connected to stability. It is based on the idea of minimizing the dissatisfaction of the most dissatisfied groups. To that aim, the concept of excess is defined.

Definition 6 Let $c \in \mathcal{C}(N)$ be a cost allocation problem and take $x \in \mathbb{R}^N$ and $S \in 2^N \setminus \emptyset$. The excess of x with respect to S, denoted by $e(S, x)$, is given by

$$e(S, x) = \sum_{i \in S} x_i - c(S).$$

The excess of x, denoted by $e(x)$, is the vector in $\mathbb{R}^{2^n-1}$ containing the excesses of x with respect to all the non-empty coalitions **disposed in non-increasing order**. More precisely, if $i \in \{1, \ldots, 2^n - 2\}$ and $e_i(x)$ denotes the i-th component of $e(x)$, then $e_i(x) \geq e_{i+1}(x)$.

Now we can provide the definition of the nucleolus in this context. The nucleolus is an allocation rule defined for cost allocation problems with a non-empty set of imputations.

Definition 7 Let $c \in \mathcal{C}(N)$ be a cost allocation problem such that $I(c) \neq \emptyset$. The nucleolus maps c to $\mathcal{N}(c) \in \mathbb{R}^N$, where $\mathcal{N}(c)$ is the unique point of the set

$$\{x \in I(c) \mid e(x) \leq_L e(y) \text{ for all } y \in I(c)\}, \quad (1)$$

$\leq_L$ denoting the lexicographic order[3] on $\mathbb{R}^{2^n-1}$. $\mathcal{N}(c)$ is said to be the nucleolus of c.

It can be check that $\mathcal{N}$ is well-defined, in the sense that the set in (1) contains in fact a unique point for those $c \in \mathcal{C}(N)$ with $I(c) \neq \emptyset$. Moreover, it can be easily proved that if $Co(c) \neq \emptyset$ then $\mathcal{N}(c) \in Co(c)$. The proof of these two features can be seen, for instance, in González-Díaz et al (2010). There are several procedures to compute the nucleolus of a cost allocation problem, but its computation can be quite hard. Maschler (1992) provides a review of those procedures. The nucleolus of the cost allocation problem in Example 1 is $(625, 725, 1650)$.

After this brief introduction to cost allocation problems, we devote the next sections of this paper to review some applications of cooperative games to cost allocation problems in a game practical environment. We have grouped these applications into three main fields, transportation, natural resources and power industry, and devote one section to each field. In every section we tackle one representative problem and explain it more carefully.

[3] According to the lexicographic order, for every $x, y \in \mathbb{R}^{2^n-1}$, $x \leq_L y$ if and only if either $x = y$ or there is $i \in \{1, \ldots, 2^n - 1\}$ such that $x_j = y_j$, for each $j < i$, and $x_i < y_i$.

3 Cost allocation and transportation

There are many applications of cooperative games in cost allocation problems related with logistics and, more particularly, with transportation. In fact, there is a large literature on game theoretical issues in combinatorial optimization problems (see, for instance, Curiel (1997)). Let us review some of those applications.

Engevall et al (1998) studied a cost allocation problem that arises in the distribution planning at a gas and oil company in Sweden. In that problem, the total distribution cost of a specific tour had to be divided among the customers that were visited. They formulated the problem as a travelling salesman game and proposed a new allocation rule that they called demand nucleolus. They compared their rule with the current tariff applied by the company and with other existing game-based rules like the Shapley value, the nucleolus, and the τ-value (see Tijs (1981)), using the real data of the gas and oil company. They concluded that the demand nucleolus is the one that provides the closest tariffs to those currently applied by the company. In order to be closer to the real problem of this gas and oil company, Engevall et al (2004) formulated the cost allocation problem as a vehicle-routing game allowing the use of vehicles with different capacities. In that paper, they mainly studied the core and the nucleolus using the real data of the company, though the Shapley value and two demand proportional rules were also computed.

Özener and Ergun (2008) also analysed the cost allocation problem that arises in a logistics network where shippers collaborate in order to negotiate better rates with a common carrier. Their main concern was to develop cost allocation rules satisfying several desirable properties. They studied the core, the nucleolus, and the Shapley value, as well as other context-specific allocation rules. Nevertheless, none of these rules satisfied all the properties posed by them. They applied their results to a data set provided by a strategic sourcing consortium for a 14 billion dollar sized industry in the United States.

Sánchez-Soriano et al (2002) studied the transport system for university students in the province of Alacant (Spain). The main aim of that paper was to distribute the cost of the system among its users, the students. First, they consider that the agents are the town councils and propose to use the egalitarian non-separable cost rule or the alternative cost avoided rule to distribute the costs among the councils (see Tijs and Driessen (1986) for details); then, the cost allocated to each council is distributed equally among its students. However, they pointed out that this approach does not take into account all the special features of the problem and they proposed an alternative approach using an aggregated egalitarian rule and a compensatory monetary system.

In the Chapter 8 of Hoang (2010) the fairness of ticket prices in public transport is analysed. He studied the case of the Dutch railway network. He formulated the ticket pricing problem as a cost allocation problem where an agent is a set of passengers of an origin-destination pair. He proposed to use a new cost allocation solution, the (f, r)-least core, which is based on the nucleolus and on the least core (Maschler et al (1979)). He also provided an algorithm to find it. He compared his results with the current tariff (based on distances) applied by the Dutch railway operator NS Reizigers.

Cooperative game theory has also been successfully applied to airline networks. One of the objectives of Changjoo (2004) is to allocate the network costs of hub-spoke networks to the users (passengers) by using cooperative game theory. He defined two models, one in which coalitions have to use the route given by the global optimal network, and other one in which coalitions can use alternative routes. He proposed to use proportional cost allocation rules and applied his models to a data set of airline passengers in the United States in 1970, obtained from the Civil Aeronautics Board.

Matsubayashi et al (2005) applies cooperative game theory to transportation of data packages in a telecommunication network. In that paper it is explored the cost allocation problem associated with hub-spoke network systems to be constructed by several agents. Its main contributions are to incorporate the congestion to the model, to study the non-emptiness of the core, to propose an allocation rule that is proportional to the flow that an agent generates, and to look for conditions such that this proportional rule lies in the core. It also studies possible extensions of the conditions for the non-emptiness of the core to a wider class of networks. Moreover, it contains an application to real data from a hub-spoke telecommunication network connecting the United States (Washington), Korea (Seoul), and Japan (Tokyo).

Transportation is also an important issue for the furniture industry. Audy and D'Amours (2008) showed that transportation coordination through collaborating planning leads to cost savings, and Audy et al (2008) studied the corresponding cost allocation problem. More specifically, Audy et al (2008) proposed a new allocation rule based on two rules existing in the literature: the equal profit rule (Frisk et al (2010)) and the alternative cost avoided rule (see Tijs and Driessen (1986) for details). Moreover it applied the new rule to a case study of four Canadian furniture companies shipping to the United States.

Another interesting issue within transportation is the design of a fair tariff system to defray the infrastructure costs. For instance, cooperative game theory was successfully applied in Villarreal-Cavazos and García-Díaz (1985) and in Makrigeorgis (1991) to a pricing problem concerning the roads in the United States. These two papers worked with the so-called generalized method, which is the nucleolus of a cost allocation problem whose agents are the types of vehicles (lights, motorbikes, trucks, etc). An application of cooperative games to the toll design for highways in Spain can be found in Mosquera and Zarzuelo (2008). In that paper, an agent is a pair of an entry and an exit point in the highway and the tolls based on the Shapley value and the nucleolus are compared with the tolls currently used.

One classical application of cooperative games in transportation cost allocation problems is Littlechild and Owen (1973). The aircrafts using the facilities of an airport pay for them and, in particular, pay a fee for every operation they perform (take-off or landing). In most cases, those fees have several components, one of them being related with the building cost of the runway used.

Littlechild and Owen (1973) proposes to design the building cost component of the fee by modelling this situation as a cost allocation problem and by using the Shapley value. More precisely, let $T = \{1, \ldots, t\}$ be the set of types of planes. The cost of a runway that is suitable for planes of type j is C_j ($j \in T$). We assume, without loss of generality, that $0 \leq C_1 \leq \cdots \leq C_t$. We denote $C_0 = 0$ and, for all $j \in T$, $a_j = C_j - C_{j-1}$. N_j is the set of movements made by planes of type j ($n_j = |N_j|$) and N is the set of all movements, i.e. $N = \cup_{j \in T} N_j$. For an $S \subset N$, we denote by $j(S)$ the largest of the types involved in the movements in S: $j(S) = max\{j \in T \mid S \cap N_j \neq \emptyset\}$. Then, define $c_a(S) := C_{j(S)} = a_1 + \cdots + a_{j(S)}$. Notice that (N, c_a) is a cost allocation problem in which $c_a(N) = C_t$ will be allocated to the movements, i.e. the fees will be constructed in order that C_t is amortized.

It is easy to see that c_a is a concave cost allocation problem. Thus, the Shapley value is a good allocation rule in this context and provides core allocations. However, it is very difficult to compute it using its formula because the number of users will be typically very large. However Littlechild and Owen (1973) provides a simple expression of the Shapley value for this class of allocation problems which is extremely helpful for its computation.

Theorem 3 *If (N, c_a) is an airport problem and $i \in N$,*

$$\Phi_i(c_a) = \sum_{j=1}^{j(i)} \frac{a_j}{n_j + \cdots + n_t}. \tag{2}$$

Littlechild and Owen (1973) illustrates its results with a set of data taken from the Birmingham airport. That paper was inspired by the works of Baker

(1965) and Thompson (1971). Later, other papers have treated these airport problems. For instance, Littlechild (1974) and Littlechild and Owen (1976) deal with the nucleolus of this class of allocation problems; Littlechild and Thompson (1977) analyses the fees policy of the Birmingham airport in the period 1968-1969 using game theory and linear programming; Vázquez-Brage et al (1997) suggests that the real agents in this problem are not the movements but the airlines, and proposes to allocate the costs using the Owen value, a variation of the Shapley value for problems with cooperation structures (see Owen (1977)). Vázquez-Brage et al (1997) illustrates its results with a set of data taken from the Santiago de Compostela airport.

Fragnelli et al (2000) analyses a cost allocation problem in a game practical environment which is related to the airport problems described above. The question in the title of the paper (how to share railways infrastructure costs?) was posed to the authors by *Ferrovie dello Stato (FS)*, the Italian national railway company in 2000, and was motivated by the reorganization of the railway sector in Europe after the approval of various European directives in 1990 and 1995. The problem is connected with the access fee that the railway transport operators must pay to the infrastructure manager for a particular journey. This fee should take into account several aspects such as the a priori profitability and social utility of the journey, congestion issues, the number of passengers and/or goods transported, the services required by the operator, infrastructure costs, etc. It is conceived in an additive way, i.e. as the sum of various fees corresponding to the various aspects to be considered.

The problem posed by FS is how to define the part of the fee which has to do with the infrastructure costs. In order to do that, Fragnelli et al (2000) proposes to model the problem as a cost allocation problem and to define the fee using the Shapley value. The paper considers that the infrastructure (at least the railways infrastructure) can be seen as a collection of facilities, each of them required by the trains at different levels of cost; so, total infrastructure costs are obtained as a sum of one-facility infrastructure costs. Now, for each facility, infrastructure costs consist of the sum of "building" costs and "maintenance" costs, the building costs being fixed and the maintenance costs being proportional to the number of trains using the facility. Notice that one-facility building cost problems are simply airport problems. So, taking into account the additivity of the Shapley value and using equation (2), in order to compute efficiently the Shapley value in this context it only remains to know how to compute the Shapley value of a one-facility maintenance cost problem. Let us formally state what a one-facility maintenance cost problem is and let us give a formula of the Shapley value for one of such problems.

Assume that $T = \{1, \ldots, t\}$ is the set of types of trains using the facility. Assume now that a train of type i has used it. We denote by C_{ij} $(i, j \in T,$ $i \leq j)$ the cost of "restore" the facility in order that a train of type j can make use of it. We assume that $0 \leq C_{ij} \leq C_{ik}$ for all $i, j, k \in T$ with $i \leq j \leq k$. We denote $m_{ii} = C_{ii}$, $m_{ij} = C_{ij} - C_{ij-1}$ for all $i, j \in T$ with $i < j$. N_j is the set of operations made by trains of type j $(n_j = |N_j|)$ and N is the set of all operations, i.e. $N = \cup_{j\in T} N_j$. Then, for $S \subset N$, define

$$c_m(S) := \sum_{i=1}^{j(S)} |S \cap N_i| C_{ij(S)} = \sum_{i=1}^{j(S)} (|S \cap N_i| \sum_{j=i}^{j(S)} m_{ij}).$$

Notice that (N, c_m) is a cost allocation problem in which $c_m(N) = \sum_{i=1}^{t} n_i C_{it}$ will be allocated to the trains using the facility. We can wonder now if these maintenance cost problems have non-empty cores. The answer is given by the following result.

Theorem 4 *If (N, c_m) is a maintenance problem, then the next four statements are equivalent:*

1. c_m is concave.

2. $Co(c_m) \neq \emptyset$.

3. $\sum_{i\in N} c_m(i) \geq c_m(N)$.

4. $m_{ij} = 0$ for all $i, j \in T$ with $i < j$.

Theorem 4 indicates that maintenance problems rarely have non-empty cores, but when they do have them, then they are even concave and their Shapley values belong to their cores. In some sense, Theorem 4 implies that, from a maintenance point of view, different types of trains should not share facilities. Obviously, they do share them in practice, because building costs have also to be considered. Next theorem provides a formula for the Shapley value of a maintenance cost problem, which allows to compute easily the Shapley value in this context.

Theorem 5 *If (N, c_m) is a maintenance problem and $i \in N$,*

$$\Phi_i(c_m) = m_{j(i)j(i)} + \sum_{r=j(i)+1}^{t} m_{j(i)r} \frac{n_r + \cdots + n_t}{n_r + \cdots + n_t + 1} + \sum_{r=2}^{j(i)} \sum_{s=1}^{r-1} m_{sr} \frac{n_s}{(n_r + \cdots + n_t)(n_r + \cdots + n_t + 1)}.$$

Fragnelli et al (2000) illustrates its results with a set of data taken from Baumgartner (1997) using a software package which computes infrastructure fees using the Shapley value. Later, Norde et al (2002) studies the core of an infrastructure cost problem, meaning by infrastructure cost problem one which is the sum of several one-facility building plus maintenance cost problems. The model and results of Fragnelli et al (2000) have also been used in Fragnelli and Iandolino (2004) to analyse a cost allocation problem related to waste transportation and disposal. Moreover Fragnelli and Iandolino (2004) provides a simple formula for computing the Owen value in this context, and applies its results to the data of the consortium "Ovadese - Valle Scrivia" (in Alessandria, Italy), comparing the Shapley value and the Owen value with the allocation rule currently used, which is proportional to the volume of waste collected.

4 Cost allocation and natural resources

The management of natural resources usually involves several agents and different tasks that need a huge financial effort, so sometimes it generates great controversies. This field makes use of models and results taken from cooperative games in order to provide cost allocations fulfilling some requirements (in Parrachino et al (2006) and Zara et al (2006) several applications of cooperative game theory in this context are discussed). Among others we find in the literature applications to the construction of multi-purpose facilities, transportation of natural resources, and environmental management. A pioneer paper in this field is Ransmeier (1942). It analysed the cost allocation problem that the Tennessee Valley Authority faced in the development of a project in the mid-southern United States. The project consisted of the construction of dams and reservoirs along the Tennessee River to generate hydroelectric power, control flooding, and improve navigation and recreational uses of the canal. In that paper the problem was not presented as a cost allocation problem, but its ideas were later reformulated by Straffin and Heaney (1981) in the language of transferable utility games with three agents (hydroelectric power, control flooding, and recreational navigation). In particular, the alternative cost avoided method allocates cost savings in proportion to each project's marginal contribution to savings. In this section we describe in detail an application in forest science and finally we briefly review some applications in other fields.

Sääksjärvi (1986) studied the cost allocation problem of timber procurement among several firms using cooperative game theory. The firms are the agents and the cost of each coalition is defined by the least-cost procurement problem for all firms in the coalition. Using data from three Finnish companies, the allocations given by the core and the nucleolus are provided. Another application in this setting appears in Frisk et al (2010). Here we describe in detail the proposed model. A firm i has to manage the flow between a finite set of supply points O^i and a finite set of demand points D^i. Each supply point is defined by a location and assortment and each demand point is defined by a location and a group of assortments. Firm i faces a planning problem in deciding which supply point should deliver to which demand point. In order to minimize the total cost a Linear Programming (LP) model can be used. Two different approaches are presented. The first one considers that only direct flows are allowed. The LP model, with variables w_{rt} representing the flow from supply point $r \in O^i$ to demand point $t \in D^i$, is given by

$$\begin{aligned} \min\ & \textstyle\sum_{r\in O^i} \sum_{t\in D^i_r} e_{rt} w_{rt} \\ \text{s.t.}\ & \textstyle\sum_{t\in D^i_r} w_{rt} \le s_r,\ r \in O^i \\ & \textstyle\sum_{r\in O^i_t} w_{rt} = d_t,\ t \in D^i \\ & w_{rt} \ge 0,\ r \in O^i,\ t \in D^i \end{aligned}$$

where for every $r \in O^i$, s_r is the supply volume and D^i_r is the set of demand points that supply point r can deliver to; for every $t \in D^i$, d_t is the demand volume and O^i_t is the set of supply points that can deliver to demand point t; and e_{rt} is the unit cost of flow between r and t, for every $r \in O^i$ and $t \in D^i$. The second approach takes into account direct flows and backhauling (Carlsson and Rönnqvist (2007)). Denoting by K^i all these routes and by x_k

the flow in route k, for every $k \in K^i$, the LP model is given by

$$\begin{aligned} \min\ & \textstyle\sum_{k\in K^i} b_k x_k \\ \text{s.t.}\ & \textstyle\sum_{k\in K^i} a_{rk} x_k \leq s_r,\ r \in O^i \\ & \textstyle\sum_{k\in K^i} d_{tk} x_k = d_t,\ t \in D^i \\ & x_k \geq 0,\ k \in K^i \end{aligned}$$

where b_k denotes the unit flow cost in route k, for every $k \in K^i$; for every $r \in O^i$ and $k \in K^i$, $a_{rk} = 1$ if route k picks up at supply point r and $a_{rk} = 0$ otherwise; and, for every $t \in D^i$ and $k \in K^i$, $d_{tk} = 1$ if route k delivers at demand point t and $d_{tk} = 0$ otherwise. Transportation cost can be decreased if several firms coordinate planning by the use of wood bartering. So the authors proposed to analyze the problem as a cost allocation problem where N represents the set of firms and the characteristic function is defined as the optimal value of an LP model. Here, two cost allocation problems are formulated, c_d and c_b, the first one associated to the case where only direct flows occur and the second one associated to the presence of backhauling. So, given a coalition S with $O^S = \cup_{i\in S} O^i$ and $D^S = \cup_{i\in S} D^i$, the characteristic function c_d assigns to S the optimal value of the LP model

$$\begin{aligned} \min\ & \textstyle\sum_{r\in O^S} \sum_{t\in D^S_r} e_{rt} w_{rt} \\ \text{s.t.}\ & \textstyle\sum_{t\in D^S_r} w_{rt} \leq s_r,\ r \in O^S \\ & \textstyle\sum_{r\in O^S_t} w_{rt} = d_t,\ t \in D^S \\ & w_{rt} \geq 0,\ r \in O^S,\ t \in D^S \end{aligned}$$

and the characteristic function c_b assigns to S the optimal value of the LP model

$$\begin{aligned} \min\ & \textstyle\sum_{k\in K^S} b_k x_k \\ \text{s.t.}\ & \textstyle\sum_{k\in K^S} a_{rk} x_k \leq s_r,\ r \in O^S \\ & \textstyle\sum_{k\in K^S} d_{tk} x_k = d_t,\ t \in D^S \\ & x_k \geq 0,\ k \in K^S \end{aligned}$$

with $K^S = \cup_{i\in S} K^i$.

Frisk et al (2010) studies the impact of coordination of eight forest firms which data are taken from a study done by the Forestry Research Institute of Sweden. Several cost instances are considered and the allocations provided by several cost allocation methods are analysed. The paper used and compared, among others, the Shapley value, the nucleolus, and the equal profit rule. This equal profit rule assigns to each cost allocation problem $c \in \mathcal{C}(N)$ having a non-empty core the set of optimal solutions of the LP model

$$\begin{aligned} \min\ & \varepsilon \\ \text{s.t.}\ & \tfrac{y_i}{c(i)} - \tfrac{y_j}{c(j)} \leq \varepsilon,\ i, j \in N \\ & \textstyle\sum_{i\in S} y_i \leq c(S),\ S \subset N \\ & \textstyle\sum_{i\in N} y_i = c(N) \\ & y_i \geq 0,\ i \in N \end{aligned}$$

All the problems examined in Frisk et al (2010) had non-empty cores; moreover, the equal profit rule provided unique allocations for all those problems. Frisk et al (2010) indicates that several firms decided to collaborate in the way described above and to use the equal profit rule.

Next we will briefly describe other applications of the cost allocation problems in the framework of natural resources. Suzuki and Nakayama (1976) considered two agricultural associations and three cities in Japan that jointly exploit the water resources. Cities can obtain water from a dam or by direct diversion from any agricultural association. Suzuki and Nakayama used the nucleolus as the allocation rule of the costs incurred from the necessary construction projects. Young et al (1982) analysed the allocation of the cost arisen of expanding the water supply system in a region of Sweden. The nucleolus and the alternative cost avoided rule were computed. Aadland and Kolpin (1998) found that the Shapley value (named there as a serial allocation rule) is one of the allocation rules that ranchers in south-central Montana use for sharing the cost of each maintenance project of the main irrigation ditch.

Loehman et al (1979) analysed the cost allocation problem of constructing a regional waste-water treatment system with eight effluent discharges along the Meramec River Basin in Missouri. The Shapley value and the generalized Shapley value (Loehman and Winston (1976)) were used as allocation rules. Dinar et al (1986) studied the cost allocation problem derived from the cooperation between three farmers and a city for the development of a regional project of waste-water treatment in order to reuse the waste-water in an irrigation water supply in the coastal plain region of Israel. The Shapley value, the nucleolus, and the generalized Shapley value were applied. Advantages and disadvantages of these allocations and the marginal cost pricing were examined. Lejano and Davos (1995) considered a water reuse project in Southern

California and computed the Shapley value, the core, the nucleolus and the normalized nucleolus in the resulting cost allocation problem.

We mention two applications of cost allocation problems arisen in environmental problems. Okada and Mikami (1992) applied cooperative game theory tools to allocate costs arisen in collective problems of pollution reduction. They compare the allocations provided by the nucleolus and the Shapley value under five different scenarios where the emission reductions were calculated using real data. Dinar and Howitt (1997) evaluated different schemes for allocation of joint environmental control costs among polluters using, as an example, the drainage pollution problem that four water districts on the west side of the San Joaquin Valley of California faced. Two different scenarios of drainage flows were considered. Among others the Shapley value, the core, and the nucleolus were analysed. The stability of the Shapley value and of the nucleolus, measured by the power index introduced in Loehman et al (1979), increases with drainage flows.

5 Cost allocation and the power industry

The power industry is undergoing a restructuring process to create a competitive open market. One of the main aims of this process is to optimize existing resources and guarantee the necessary investments to satisfy future electricity demands at reasonable rates. This restructuring process is based on the decentralization of the power industry business into three separate businesses: generation, transmission, and distribution. While the generation business usually operates in a competitive environment transmission and distribution are, in general, monopoly markets. Hence, regulation seems to be the only way to introduce the effect of competition.

Embedded costs of power networks represent a large part of the total costs. So, the development of a fair and stable network pricing model is needed. Mainly, the devised cost allocation methods are based on the patterns of individual usage of the network (Sood et al (2002)). Due to the complexity of these methods, it is difficult to identify the fairest one; moreover, these usage-based methods may not provide stable solutions.

The upcoming market has multiple agents using the network: loads and generators. This has induced the use of cooperative game theory for allocating the network embedded costs. The nucleolus and the Shapley value have been the most used cost allocation methods. Next, we show how cooperative games could be apply to cost allocation problems in the electricity market by summarizing the paper Zolezzi and Rudnick (2002). Later on, we revise the existing literature about this topic.

Zolezzi and Rudnick (2002) proposed a cost allocation method for charging the transmission cost of an electric network among its users. The method, which applies cooperative games methodology, is based on the transmission network capacity used by consumers. Let us take a finite set of time periods $T = \{1, \ldots, t\}$, and an electric network with a finite set of buses[4] $M = \{1, \ldots, m\}$ and a finite set of transmission lines $K = \{1, \ldots, k\}$. At

[4] In this literature the word *bus* is commonly used for referring to a node in a network.

each bus $q \in M$ a generator and/or a consumer are located. For every time period $r \in T$, each bus $q \in M$ has the following associated parameters: the maximum power $g_{q,r}^{max}$, the minimum power $g_{q,r}^{min}$ and a generation variable cost α_q describing the generator; a load L_q^r describing the consumer (if there is no generator or no consumer, we assume the value zero for the corresponding parameters). The susceptance of each transmission line $p \in K$ is given by $1/\beta_p$. It is assumed that there is no congestion and that the network is adequately planned in terms of the capacity, quality, security, and backup. It is possible to associate megawatts of flow with costs, so the cost an agent or a coalition faces at each line can be measured by the flow that goes through it. Fix a time period $r \in T$. Under these parameters and using the DC flow approximation (a standard method in this literature to approximate the power flow in a network, see Wood and Wollenberg (1996)), the flow for every line is obtained as

$$F^r = bA\Theta^r \tag{3}$$

where $b = diag(1/\beta_1, \dots, 1/\beta_k)$, A is the network adjacency matrix given by

$$a_{pq} = \begin{cases} 1 & \text{if line } p \text{ exits bus } q \\ -1 & \text{if line } p \text{ enters bus } q \\ 0 & \text{otherwise} \end{cases}$$

and $\Theta^r \in \mathbb{R}^m$ is the bus voltage angle vector, given by $\Theta = B^{-1}P^r$ where:

- the matrix B is equal to $A^t bA$, and

– the real power injection vector $P^r \in \mathbb{R}^m$ is defined by $P^r_q = g_{q,r} - L^r_q$, for all $q \in M$, the variable $g_{q,r}$ representing the power generation at bus $q \in M$ and at time period $r \in T$.

To determine the real power injected generations, an economic dispatch is performed using the generation variable cost α_q and the load requirements L^r_q, for every $q \in M$; that is, an optimal solution of the following LP problem is obtained

$$\begin{aligned} \min\ & \textstyle\sum_{q\in M} \alpha_q g_{q,r} \\ \text{s.t.}\ & \textstyle\sum_{q\in M} g_{q,r} = \sum_{q\in M} L^r_q \\ & g^{min}_{q,r} \le g_{q,r} \le g^{max}_{q,r},\ q \in M. \end{aligned}$$

Once the above LP problem is evaluated and the flow $F^r \in \mathbb{R}^m$ is obtained for every $r \in T$, the maximum flow condition is determined for each line $p \in K$, that is, r^{max}_p such that $F_p^{r^{max}_p} = \max\{F^r_p \mid r \in T\}$. Consumers may act forming coalitions if it is beneficial for them. They will cooperate if no agent or coalition of agents will have a cost greater than its stand-alone cost, and the resultant cost allocations will cover all transmission costs. It is assumed that there is open access and, so, consumers may establish commercial contracts with any generator. Here, cooperation is analyzed for each transmission line at a time under its maximum flow condition. Take $N = \{1, \ldots, n\}$ the set of consumers, a transmission line $p \in K$, and a coalition $S \subset N$. We denote by S_p the set of consumers in S that use line p in order to satisfy their demand. Under the maximum flow requirement of line $p \in K$, coalition S faces the economic dispatch problem

$$\begin{aligned} &\min \textstyle\sum_{q\in M} \alpha_q g_q \\ &\text{s.t. } \textstyle\sum_{q\in M} g_q = \sum_{q\in S_p} L_q^{r_p^{max}} \\ &\quad g_{q,r_p^{max}}^{min} \le g_q \le g_{q,r_p^{max}}^{max},\ q\in M \end{aligned}$$

which allows the computation of the flow through the network, $F^S \in \mathbb{R}^m$, using Expression (3). The cost associated to coalition S is given by $c_p(S) = F_p^S$. Fix an allocation rule ϕ and apply it to every cost allocation problem (N, c_p) obtaining a vector of payments $\phi^p = (\phi_1^p, \ldots, \phi_n^p)$. Finally, the total cost of a transmission system to each consumer i is given by

$$\sum_{p\in K} \frac{\max\{0, \phi_i^p\}}{\sum_{j\in N} \max\{0, \phi_j^p\}} d^p$$

where $d^p \in \mathbb{R}$ is the real total cost of transmission line $p \in K$. The method does not consider compensation for negative values. An application of this method to the main Chilean system appears in Zolezzi and Rudnick (2003). The authors considered a simplified eight bus model and used this method taking the Shapley value as an allocation rule. They compare the results obtained in that way with the results arisen using the generalized load distribution factors (Rudnick et al (1995)) and the average participation method (Bialek (1996)).

Next we provide a review of some papers containing applications of cooperative games to cost allocation problems arising in the electricity market. Hobbs and Kelly (1992) studied the usage of game theory to analyse electric transmission pricing policies in the United States. This study deals with both non-cooperative games and cooperative games. The cooperative analysis is based on the ideas of the core and on a new solution concept that they called policy restricted core. They applied these solution concepts to eight interconnected electric utilities in the eastern United States.

Tsukamoto and Iyoda (1996) analysed the usage of cooperative games for fixed-cost allocation to wheeling transactions in a power system. In their framework, each wheeling transaction is considered as a player in the game. They used the nucleolus for this purpose and compared it with other allocating methods traditionally used in the economics literature. Asano and Tsukamoto (1997) reviewed the transmission pricing problem in Japan. Among other recommendations to improve the Japanese pricing system, they proposed the application of the cooperative games discussed in Tsukamoto and Iyoda (1996).

Yu et al (2001) presented two methods for transmission embedded cost allocation based on the nucleolus and on the Shapley value, respectively. Again, the wheeling transactions are the players. Yu et al (2001) applied these rules to some case studies and concluded that they reflect the quality of transmission service by giving different price signals to transactions.

In Tan and Lie (2002) a different approach to the problem of transmission network cost allocation is proposed. Here, a cooperative game approach is also used, but the cost has to be allocated among the loads in the network instead of among the transactions. Consequently, the players in this cooperative game are the loads. They proposed the Shapley value to allocate the transmission costs. They compared it with some conventional allocation rules (postage-stamp, MW-mile, etc) and found that the Shapley value solves some drawbacks of the conventional rules, such as the failure in the cost recovery. However, they pointed out that the Shapley value is not widely used in practice since the data requirements for real world problems quickly become too cumbersome to deal with.

Yu et al (2003) again dealt with the problem of allocating the transmission embedded cost among the transactions instead of among the loads. They used the core, the nucleolus and the Shapley value for the allocation of transmission network embedded cost recovery based on capacity-use and reliability benefit patterns. As some of the case studies shown, the Shapley value for the class of games defined in this paper can lie outside the core. Stamtsis and Erlich (2004) followed the same line and analysed the cost allocation problem for the fixed cost of a power system through the core, the nucleolus and the Shapley value. They concluded that the Shapley value is preferable when it lies in the core of the game.

Bjørndal et al (2005) also studied the problem of allocating the transmission embedded cost among the transactions. They proposed a new method for allocating the embedded costs combining some conventional usage-based methods with the ideas under the nucleolus.

Bhakar et al (2009) studied the new restructured power market to allocate the network embedded cost among generators and loads. They used the nucleolus and the Shapley value for that purpose. All the former studies are based on peak loading by the agents, but in a real network, some agents may not be connected to the system for some time. Bhakar et al (2010) took into account this inherent probabilistic nature of the agents and provided probabilistic cost allocation methods based on the nucleolus and on the Shapley value.

Azevedo et al (2009) analysed the use of cooperative games to allocate the network cost that is not recovered by the application of short-term marginal prices. They compared the nucleolus and the Shapley value with two commonly used allocation methods by applying them to a six bus Garver network (see Villasana et al (1985) for details on Garver networks).

Songhuai et al (2006) used the nucleolus and the Shapley value to allocate the transmission loss costs in a bilateral electricity market. In that paper, the bilateral transactions are considered to be the players. They tested the two rules in several systems and concluded that the nucleolus performs well for the market principles of openness, equality, and impartiality. For solving the problem of calculating the nucleolus for large-scale systems they proposed to use parallel computing. In a related work, Lima et al (2008) analyzed the same problem but in a slightly different context; they dealt with pool-based electricity instead of with bilateral electricity. In their model the equivalent bilateral exchanges are considered to be the players in the cooperative model. They searched for allocations in the core, but they realized that it can be empty. Then, they studied other allocation rules taken from cooperative games, such as the Shapley value, the bilateral Shapley value (see Ketchpel (1995)) and the kernel (see Davis and Maschler (1965)). Moreover, they found that forming a coalition is not always the best option for the players and, thus, they studied the coalition formation process.

Apart from the embedded transmission costs and transmission loss costs, there are other issues in a power network which also generate costs, like the ancillary services or the transmission line expansions. Cooperative games were also successfully applied to allocate these costs. Evans et al (2003) used the kernel to allocate the costs incurred for the expansion of electrical transmission systems. Erli et al (2005) proposed the use of the core and of the nucleolus to analyse the cost allocation problem related to the costs incurred by an expansion of the transmission line to tackle with congestion. In Pan et al (2007), the issue of pricing the services provided by power system stabilizers is studied and they propose to use the nucleolus. In Xie et al (2008), the nucleolus and the Shapley value are applied to solve the peaking cost allocation problem of hydro-thermal power systems. They also applied the allocation rules to a real case in the Northwest China power system.

6 Concluding remarks

Cooperative game theory is an important field within game theory, which is concerned with groups of players willing to allocate the joint benefits of their cooperation or, equivalently, the joint (reduced) costs when they cooperate. Although its impact in economic theory is smaller than the impact of other areas of game theory, it is probably the chapter of game theory with more applications in real-world problems (jointly with auction theory). In fact, it is nearly impossible to survey in a single paper the main practical applications of cooperative games.

In this work we review the applications of transferable utility games in the areas of transportation, natural resources and power industry. We have left other models within cooperative games (such as bargaining models, non-transferable utility games, non-atomic games, etc), other fields of application (such as benefit allocations, voting, bankruptcy, etc), and other areas. We have found that the models, concepts and results of transferable utility games are commonly used in practical cost allocation, and that this is a very active field nowadays.

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